CHE 3804 NUCLEAR ENGINEERING

SECTION 5

REACTOR KINETICS

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Reactor kinetics is the study of how neutron power changes with time. It is usually associated with short term changes initiated by natural perturbations or imposed transients. Control systems have to be designed to maintain the desired neutron power following both types of short term changes.

In the longer term there are changes due to the build-up of neutron absorbers and the burn-up of neutron producers in the core of the reactor. Since these changes are slow they do not affect the control system as such but the overall reactor configuration has to be adjusted to maintain the desired balanced condition for a steady neutron chain reaction. The control system then maintains equilibrium about this balanced condition.

Both short term and long term effects are considered in this section with the initial emphasis on the short term effects commonly falling within the scope of reactor kinetics.

NEUTRON LIFETIME

An important consideration in reactor kinetics is the extremely high velocity of neutrons (2200 m/s when thermalised at 20°C). With the high probability of neutron scattering during moderation from high energies and of neutron capture to cause fission at thermal energies and the extremely close spacing of fissile nuclei in the fuel neutrons do not travel very far in the reactor. The overall neutron lifetime during thermalisation and diffusion from production due to fission until absorption to produce fission is thus extremely short. The average neutron lifetime in a CANDU reactor is about 1 millisecond. The complete neutron cycle from one generation to the next takes only this amount of time. It is evident therefore that any small deviation from the equilibrium situation, where the number of neutrons in one generation is equal to that of the previous generation, will very rapidly grow in magnitude.

If all neutrons had a lifetime of about 1 millisecond it would be almost impossible to design a control system that would be able to sense changes and effect control before the neutron

population grew or shrank out of the control range. Fortuitously some neutrons are produced a short time after the actual fission process. These are known as *delayed neutrons*. The kinetic behaviour of a reactor is critically dependent upon the existence of these delayed neutrons as they have the effect of increasing the average lifetime of the neutrons arising from fission. This increase in average lifetime to about 1 second due to less than 1% of the neutrons having much longer effective lifetimes enables control systems to maintain stable operation provided that certain limits are not exceeded.

REACTOR POWER

Reactor power P is given by the following formula where ϕ is the neutron flux, Σ_f the macroscopic fission cross section, V the volume of the reactor and E_f the energy released per fission in joules.

$$P = \phi \, \Sigma_f \, V \, E_f$$

Note that the neutron flux ϕ and the macroscopic fission cross section Σ_f are given by:

$$\phi = n \, v$$

$$\Sigma_f = N \, \sigma_f$$

Here n is neutron density, v the neutron velocity, N the nuclei density (of fissile material) and σ_f the microscopic cross section.

For a given reactor configuration it is evident that the reactor power P is proportional to the neutron flux ϕ and to the neutron density n since the other factors in the equations are constant in the short term. Any variation in neutron density will therefore be reflected as a variation in reactor power. Although the reactor kinetics equations are related to variations in neutron density they are often represented directly as variations in reactor power and the two terms are used synonymously.

In the control room of a nuclear reactor there are instruments which indicate the neutron flux levels in the reactor but ultimately it is the reactor power level which is monitored and controlled. Hence the equations presented below are written in terms of reactor power.

BASIC REACTOR KINETICS

The basic reactor kinetics equation (without delayed neutrons) may be derived by using the definition of the neutron multiplication factor k.

k = neutrons in one generation / neutrons in previous generation

Consider a slight but steady increase in the number of neutrons in successive generations, that is, k is greater than unity. If the initial neutron density is n then, after one generation, the neutron density will be equal to kn and the change in neutron density Δn will be given by:

$$\Delta n = kn - n$$

If the neutron lifetime is ℓ then this change from one generation to the next will take place in time ℓ that is the neutron lifetime ℓ is equal to the time interval Δt and the equation can be written as:

$$\Delta n / \Delta t = (k\,n - n) / \ell$$

Substitution into this equation and conversion to differential form, assuming that k is greater than unity and that Δk is its deviation from unity, gives

$$dn/dt = \Delta k\, n / \ell$$

This equation can be integrated from time zero to time t and converted to exponential form as follows:

$$\int_o^t dn / n = \int_o^t (\Delta k / \ell)\, dt$$

$$\mathrm{Ln}\,(n_t / n_o) = (\Delta k / \ell)\, t$$

$$n_t = n_o\, e^{(\Delta k/\ell)t}$$

Here n_t is the neutron density at time t while n_o is the initial neutron density at time zero. Since reactor power P is proportional to neutron density n this equation may be written as:

$$P_t = P_o\, e^{(\Delta k/\ell)t}$$

This is the basic reactor kinetics equation assuming that all neutrons have the same lifetime ℓ.

REACTOR PERIOD

Reactor period τ is defined as the time taken for the reactor power to increase by a factor of e. This is a common way of specifying a rate of power increase in a nuclear reactor. After one reactor period the time t will be equal to τ and the reactor power P_t after time t will be equal to the initial reactor power P_o multiplied by a factor of e.

$$t = \tau$$

$$P_t = P_o\, e$$

If these values are substituted into the basic reactor kinetics equation the following is obtained:

$$P_o\, e = P_o\, e^{(\Delta k/\ell)\tau}$$

$$\tau = \ell / \Delta k$$

Thus the reactor period may be obtained directly from the neutron lifetime ℓ and the change in the value of the neutron multiplication factor k.

DOUBLING TIME

Reactor *doubling time* is the time taken for the reactor power to rise to twice its initial condition. This is used during reactor commissioning and also during reactor start-up particularly following refuelling.

Fuel *doubling time* is the time taken for a breeder reactor to produce twice as much fissile material as was initially in the reactor. This occurs due to the absorption of excess neutrons in U-238 and its conversion to Pu-239 while the U-235 is consumed to produce power.

It is seen from the above that an important parameter in reactor kinetics is Δk where Δk is the change in the neutron density. Generally in an operating reactor k is very nearly unity (exactly unity at a steady power level) and Δk is a small fraction. Δk thus represents a departure from the steady state condition in a positive or negative direction and is commonly known as positive or negative *reactivity*. Very conveniently reactivity has been given units

of k (British and Canadian terminology) although it is really dimensionless. This allows the use of mk for small values of reactivity where 1 mk is equal to 0.001 k. For 1 mk of reactivity Δk would be 0.001 and k would be 1.001. Many text books use an alternative terminology where reactivity is expressed in dollars and is related not to the steady state condition but to the margin between the steady state condition and the prompt critical condition which will be defined later.

REACTOR KINETICS WITH DELAYED NEUTRONS

Following the fission process some neutrons are emitted somewhat later than those produced at the time of fission. These are *delayed neutrons* as opposed to the normal *prompt neutrons* which appear immediately. The delayed neutrons arise from some twenty fission products or *delayed neutron precursors* which are themselves unstable. In decaying they emit neutrons to bring them closer to the stability curve. For convenience these precursors are grouped into six groups each with a characteristic half-life. The first group has a half-life of 55 seconds while the last group has a half-life of only 0.2 second. Most of the delayed neutrons are however from intermediate groups having half-lives between these extremes. Depending upon the fuel used, the total fraction of delayed neutrons and the average half-life of the delayed neutron precursors will vary. For Uranium-235 the fraction of delayed neutrons β is 0.0065 or approximately 0.7% while their average half-life is about 7 seconds giving a decay constant λ of about 0.1 per second.

This has a profound effect on the kinetics of a reactor since the average life of the delayed neutrons is about 10 seconds. Although only 0.7% of the total fission neutrons are delayed neutrons their average life is some ten thousand times that of the prompt neutrons. There will thus be some ten thousand generations of prompt neutrons before the effects of the delayed neutrons are felt. This gives time for the reactor control system to sense changes in the neutron density and take corrective action. It is of fundamental importance to the safety of the reactor that the neutron chain reaction does not diverge under the influence of prompt neutrons only. Such a situation is known as *prompt critical* and leads rapidly to destruction of the reactor due to an excess power level.

In order to understand the effect of delayed neutrons it is convenient to set up a hypothetical model in which the delayed neutrons are assumed to be produced at the same time as the prompt neutrons but are then held up in a bank for a period of time before being released to join the prompt neutron cycle. Thus if there are n neutrons in one generation there will be kn in the next generation. Of these $kn\beta$ will be delayed neutrons and $kn(1 - \beta)$ will be prompt neutrons. The $kn\beta$ delayed neutrons will only be added to the cycle after many

subsequent prompt neutron generations by which time the number of prompt neutrons will have changed considerably from the initial kn(1 - β).

An equation representing this situation may be derived from the previous equation representing the variation of neutron density with time (without the effect of delayed neutrons).

$$\Delta n / \Delta t = (k\,n - n) / \ell$$

In differential form this becomes:

$$dn/dt = (k\,n / \ell) - (n / \ell)$$

If the delayed neutrons are separated to be stored in a precursor bank then the equation for prompt neutrons becomes:

$$dn/dt_{prompt} = (k\,n / \ell)\,(1 - \beta) - (n / \ell)$$

The delayed neutrons appear later due to the decay of the precursors in the bank. If c is the concentration of the precursors with decay constant λ then:

$$dn/dt_{delayed} = \lambda\,C$$

Combining these equations for prompt and delayed neutrons gives for the neutron cycle:

$$dn/dt = (k\,n / \ell)\,(1 - \beta) + (\lambda\,C) - (n / \ell)$$

Considering separately the equation for the precursor bank where delayed neutrons are added by fission and removed by decay:

$$dC/dt = (k\,n\,\beta / \ell) - (\lambda\,C)$$

These two differential equations for the neutron cycle and the precursor bank may be solved simultaneously. For small values of Δk, that is, for k close to unity the solution is as follows:

$$n_t = n_o\,[\{\beta / (\beta - \Delta k)\}e^{\{\lambda\Delta k/(\beta-\Delta k)\}t} - \{\Delta k / (\beta - \Delta k)\}e^{-\{(\beta-\Delta k)/\ell\}t}]$$

After a short period of time the second term goes to zero and can therefore be neglected.

$$n_t = n_o \{\beta / (\beta - \Delta k)\} e^{\{\lambda \Delta k/(\beta - \Delta k)\}t}$$

Furthermore since reactor power is proportional to neutron density the above equation reduces to :

$$P_t = P_o \{\beta / (\beta - \Delta k)\} e^{\{\lambda \Delta k/(\beta - \Delta k)\}t}$$

This is known as the prompt jump approximation. If numerical values such as those given below are inserted into the complete equation an expression in terms of time only is obtained:

Δk	$= 0.001$	(reactivity change)
β	$= 0.007$	(delayed neutron fraction)
λ	$= 0.1\ s^{-1}$	(precursor decay constant)
ℓ	$= 0.001$ s	(prompt neutron lifetime)

$$P_t = P_o [1.17\ e^{0.0167\,t} - 0.17\ e^{-6.0\,t}]$$

If the variation of each term is plotted separately and then added together it is evident that the second term decays to zero within one second while the first term has a step before the start of the exponential rise. The second term eliminates the step only and provides a smooth transition into the exponential rise. By eliminating the second term the initial transient is a step instead of a ramp and the so called *prompt jump approximation* is obtained.

Analysis of the prompt jump approximation shows that there is a similarity with the basic reactor kinetics equation.

$$P_t = P_o\, e^{(\Delta k/\ell)t}$$

This has no step prior to the exponential rise but if the neutron lifetime ℓ can be modified to include the delayed neutrons an exponential rise of the appropriate magnitude can be obtained. This average neutron lifetime can be obtained by setting the exponents equal to one another

$$\Delta k / \ell_{ave} = \lambda\, \Delta k / (\beta - \Delta k)$$

$$\ell_{ave} = (\beta - \Delta k) / \lambda$$

Thus the *average neutron lifetime approximation* is obtained:

$$P_t = P_o e^{(\Delta k/tave)t}$$

This approximation neglects the initial rapid rise in power following a small positive step change in reactivity.

REACTOR KINETICS NUMERICAL MODEL

It is instructive to develop a simple numerical model to illustrate reactor kinetics. In order to obtain numerical simplicity and rapid changes from one generation to the next excessively large values for the various parameters have been chosen. These are given below and it is assumed that the reactivity of the reactor is increased by Δk, that is, the value of k is suddenly increased from the steady state value of unity

$\Delta k = 0.05$ (that is $k = 1.05$)
$\beta = 0.1$ (instead of 0.007)

The number of neutrons N in the first generation is 1000. Prior to the increase in reactivity the number of prompt neutrons $kN(1 - \beta)$ in successive generations is 900 and the number of delayed neutrons $kN\beta$ in successive generations is 100. After the increase in reactivity the number of delayed neutrons $kN\beta$ entering the precursor bank is 105 while the number of prompt neutrons $kN(1 - \beta)$ entering the neutron cycle is 945. The number of delayed neutrons entering the neutron cycle from the precursor bank remains at 100 for many generations until the excess begin to be released after the delay period.

In the next generation the total number of neutrons is 1045. This produces 110 delayed neutrons and 988 prompt neutrons to give 1088 neutrons in the following generation and 1128 in the generation thereafter.

It is seen from this that the total number of neutrons rises rapidly at first but that the rate of rise diminishes to produce a curve similar to that predicted by the mathematical equations.

After a period of time and many generations the curve will level off at a value close to that predicted by the prompt jump approximation that is 210 delayed neutrons entering the precursor bank and 1890 prompt neutrons entering the neutron cycle from an initial 2000 neutrons. Note that the prompt jump factor is 2. By this time some excess delayed

neutrons will be beginning to leave the precursor bank driving the number of delayed neutrons above the initial 100. As this number grows above 110 the total number of neutrons will rise above 2000 and the exponential rise in neutron density will commence.

It is evident from the above that it is the delayed neutrons which hold back the rise in neutron density. If however the value of Δk is increased so that it is greater than the value of β then the number of prompt neutrons being produced will exceed the total number of neutrons in the previous generation and the neutron density will rise regardless of the effect of the precursor bank. This condition is known as *prompt critical* and is a highly unstable and rapidly diverging condition. In this condition the reactor is critical with k equal to unity on prompt neutrons only.

NEGATIVE REACTIVITY

If there is a decrease in power due to k being less than unity, that is Δk is negative, the same equations as above still apply. In the prompt jump approximation there will be a prompt drop so that the power immediately after the drop P_1 will be given by:

$$P_1 = P_o \{\beta / (\beta - \Delta k)\}$$

The power after the prompt drop will decrease exponentially with a period τ given by the same formula as before:

$$\tau = (\beta - \Delta k) / (\lambda \Delta k)$$

The power would then decrease progressively to the desired value with the power at time t equal to:

$$P_t = P_1 e^{\{\lambda \Delta k/(\beta - \Delta k)\}t} \qquad (\beta > \Delta k)$$

If the reactor has to be shut down quickly in an emergency or accident the value of Δk will be large due to the sudden insertion of reactor control devices which absorb neutrons strongly or the activation of control systems which make the reactor assembly no longer able to sustain a nuclear chain reaction. In the event of Δk being large and negative such that the absolute value of Δk is very much greater than that of β the period τ becomes:

$$\tau \approx + \Delta k / (1 - \lambda \Delta k)$$

$$\tau \approx -1 / \lambda$$

The power after the prompt drop would then decrease rapidly to the shutdown level with the power at time t equal to:

$$P_t = P_1 e^{-\lambda t} \qquad (\Delta k >> \beta)$$

In the above λ is the decay constant of the delayed neutron precursors.

The two formulae given above for the power at time t are obviously different. The difference between them however is less than 1% if the absolute value of Δk is greater than 0.020 that is if more than 20 mk of negative reactivity is inserted into the reactor. This is usually the case in the event of an emergency shutdown also known as a *reactor trip* or *reactor scram*. It is important to note that no matter how fast the reactor trip the power will take a finite time to decay due to the effect of the delayed neutrons.

SHUTDOWN CONDITIONS

The delayed neutrons also influence the reactor under shutdown conditions. The six delayed neutron precursor groups have different half-lives and different yields. The first group has a relative yield of only 4%, that is, its yield is 4% of the delayed neutron fraction of 0.007 but it has a half-life of 55 seconds. By plotting the decay curves of the various groups assuming a given number, say 1000, of delayed neutrons it is evident that after about 80 seconds the first group dominates and after twice that time is the only group of any consequences still producing delayed neutrons. While the reactor is shut down this decay process continues and some neutrons are always present since the decay curve is asymtotic.

Both Uranium-235 and Uranium-238 are naturally unstable and decay mainly by α-particle emission. Both also fission spontaneously giving off neutrons in the process. Any fuel, even unused fuel, produces small numbers of neutrons. Highly enriched Uranium-235 has the potential of initiating a runaway neutron chain reaction from these free neutrons if a critical mass is created. When storing nuclear fuel therefore precautions must be taken to avoid creating a critical configuration. This is particular important when handling highly enriched fuel for research reactors or nuclear weapons.

Furthermore in heavy water moderated reactors, such as the CANDU, neutrons can be created by the interaction of high energy γ-rays with the deuterium atoms:

$$^{2}H_{1} + {}^{0}\gamma_{0} = {}^{1}H_{1} + {}^{1}n_{0}$$

The γ-rays must have an energy greater than 2.2 MeV. Certain fission products produce γ-rays with energies in excess of this value. The half-life of these fission products is 15 days. After a reactor has been shut down neutrons continue to be produced from this reaction for a considerable period.

There are therefore three main sources of neutrons within a nuclear reactor after shutdown:

Spontaneous Fission

Uranium-235:	0.3 disintegrations / second per kg
Uranium-238	6.9 disintegrations / second per kg

Decay of Delayed Neutron Precursors

Delayed neutron fraction	= 0.007
Relative yield of Group 1	= 0.04
Half-life of Group 1	= 55 seconds

Photo-neutron Emission

$^{2}H_{1} + {}^{0}\gamma_{0} = {}^{1}H_{1} + {}^{1}n_{0}$
γ-ray energy > 2.2 MeV
Half-life of γ-ray producers = 15 days

SOURCE MULTIPLICATION

From the neutron sources described above there are always a number of stray neutrons being produced in a nuclear reactor even when shut down. These neutrons can cause fission in fissile fuel thus producing more neutrons and establishing a chain reaction. When the reactor is shut down the neutron multiplication factor k is less than unity.

This means that the chain reaction will decay but in the meantime other stray neutrons will start new chain reactions. The result is that, along with the stray neutrons, there will fission neutrons so that the total number of neutrons will be greater than would arise from the neutron sources only. The factor by which the total number of neutrons is greater than the number of source neutrons is known as the *subcritical multiplication factor.*

The subcritical multiplication factor can be determined by imagining that the source contributes a fixed number of source neutrons to each generation of fission neutrons. Source neutrons are continually added but fission neutrons are continually lost due to the value of k being less than unity. Ultimately a balance is reached with a steady number of neutrons S_∞ somewhat greater than the number of source neutrons S_0. The relationship between these and the value of k is given by

$$S_\infty = S_0 / (1- k)$$

$$S_\infty = - S_0 / \Delta k$$

Here S_∞ is the number of neutrons per second measured by instrumentation in the reactor and S_0 is the number of neutrons per second being produced from the neutron sources. S_∞ is equivalent to the neutron power of the reactor.

RATE LOG POWER

Neutron power in a nuclear reactor is usually measured on a logarithmic scale since the range of neutron density from shut down to full load conditions spans several decades. The rate log is defined as the change in log power per unit time. This is equal to the inverse of the reactor period τ.

$$\text{Rate Log} = 1 / \tau$$

Usually the rate log is represented by logarithms to the base 10 while the rate log determined from the period is given by logarithms to the base e. Within the instrumentation circuitry therefore an appropriate conversion must be made by applying a factor of 2.3026.

POWER TRANSIENTS

It is evident that the measured neutron power is greater than the source neutron power by the subcritical multiplication factor since neutron power P is proportional to number of neutrons per second.

$$P_\infty = P_0 / (1 - k)$$

$$P_\infty = - P_0 / \Delta k$$

Under subcritical conditions k is less than unity and Δk is negative. As Δk becomes smaller the subcritical multiplication factor becomes larger and the number of neutrons grows. Also the number of generations for the number of neutrons to stabilize at a new value increases so that the neutron power rises rapidly at first and then levels off at the new equilibrium value. When the reactor becomes critical the neutron power does not level off but goes on increasing since k is unity and S_o is finite in value. In a real nuclear reactor S_o is so small as to be insignificant and perturbations cause the control system to vary k slightly above and below unity.

When a reactor is tripped and held in the shut down condition with a significant value of negative reactivity the neutron power decays initially according to the relations previously developed:

$$P_1 = P_o \{\beta / \beta - \Delta k)\} \qquad (P_o = \text{initial reactor power})$$

$$P_t = P_1 e^{-\lambda t}$$

This continues only up to the point where the power is equal to the source neutron power multiplied by the subcritical multiplication factor:

$$P_\infty = P_o / (1 - k) \qquad (P_o = \text{source neutron power})$$

From this point on the reactor power is governed by the decay of the source neutrons which in a CANDU reactor is governed by the half-life of the high energy γ-ray emitting fission products. The difference between P_∞ and P_o is determined by the value of k.

Since a CANDU reactor has a strong source of photo-neutrons the value of P_∞ remains sufficiently high to be measured on the reactor instrumentation for about three months after being put into the shut down condition. This facilitates restarting of the reactor. In other reactors which do not use heavy water the neutron flux very soon falls below the threshold at which it can be read by the instrumentation. In these reactors a start-up source is used to permit proper monitoring of the neutron flux. A suitable start-up source can be made using Beryllium-9 and an isotope giving off γ-rays with a strength greater than 1.6 MeV.

$$^9Be_4 + {}^0\gamma_o = {}^8Be_4 + {}^1n_o$$

Such sources are inserted into nuclear reactors on start-up, particular the initial start-up, to ensure that a high enough neutron flux is obtained to be able to be read on the reactor instrumentation.

DECAY HEAT

The neutron power after shutting down the reactor produces some heat. After a very short period of time however significantly more heat is produced from the fission products. As these decay the heat production decreases but remains an important factor for a long time afterwards. For a 600 MW (electrical) CANDU unit such as Point Lepreau about 1 MW of heat would be produced after about a year in the shut down condition.

XENON TRANSIENTS

In a nuclear reactor during the fissioning of the fuel a range of fission products is produced. The relative yield of each of these fission products varies with the type of fissile material and the energy of the neutrons. Generally Uranium-235 is fissioned by thermal neutrons and it is the relative yield of fission products from this reaction that dominates. Of the many fission products that are produced most are unstable and decay to other nuclides. Most of these daughter nuclides are not neutron absorbers since they have decayed from fission products with an excess of neutrons. A few however end up as strong neutron absorbers and this is detrimental to the continuation of the fission chain reaction. Xenon-135 is the most important neutron absorbing nuclide produced as a result of the fission process. Xenon-135 is produced directly as a fission product and also as a daughter of Iodine-135, another fission product. Xenon-135 being a strong neutron absorber soon transmutes to Xenon-136 a stable nuclide which is a very weak neutron absorber.

The result of the above in an operating reactor is the production of Iodine-135 and Xenon-135 due to the fission process, the decay of Iodine-135 to Xenon-135 by β-particle emission, the burnup or transmutation of Xenon-135 to Xenon-136 by neutron absorption and the decay of Xenon-135 to Cesium-135 by β-particle emission. This leads to various transients determined by the decay and burnup rates of the different nuclides. The burnup rate in turn depends upon the neutron flux level or reactor power level. This complicated process can however be analysed mathematically if the decay constants, relative yields and cross sections are known. These values are fixed and well known:

For Iodine-135:	fission yield	γ_I	=	0.063
	half-life	$t_{1/2}$	=	6.7 h
	decay constant	λ_I	=	0.104 h^{-1}
	decay constant	λ_I	=	2.87 x 10^{-5} s^{-1}
For Xenon-135:	fission yield	γ_{Xe}	=	0.003

half-life	$t_{1/2}$	=	9.2 h
decay constant	λ_{xe}	=	0.075 h^{-1}
decay constant	λ_{xe}	=	2.09 x 10^{-5} s^{-1}
cross-section	σ_{xe}	=	3.5 x 10^{6} b

The mathematical equations may be derived by considering the build-up and decay of both Iodine-135 and Xenon-135 and noting that the decay of the former contributes to the build-up of the latter.

The Iodine-135 builds up from its own fission yield and decays according to its half-life. The rate of change of Iodine-135 is thus given by:

$$dN_I/dt = \gamma_I \Sigma_f \phi - \lambda_I N_I$$

Here the macroscopic cross section Σ_f and the neutron flux ϕ give the total number of fuel nuclei fissioned. Multiplying this by the fission yield γ_I gives the number of Iodine-135 nuclei produced.

The Xenon-135 builds up from its own fission yield in the same way as the Iodine-135 and also from the decay of Iodine-135. It decays according to its half-life and is also burned up by the neutron flux ϕ. The rate of change of Xenon-135 is thus given by:

$$dN_{Xe}/dt = \gamma_{Xe} \Sigma_f \phi + \lambda_I N_I - \lambda_{Xe} N_{Xe} - \sigma_{a\,Xe} N_{Xe} \phi$$

Here the microscopic cross section σ_a multiplied by the neutron flux ϕ and the number of nuclei N_{Xe} gives the number of nuclei destroyed. For a typical operating reactor the dominant terms are $\lambda_I N_I$ and $\sigma_{aXe} N_{Xe} \phi$ showing that most Xenon is derived from decay of Iodine-135 and lost by burnup due to the neutron flux.

At equilibrium when the rate of change of each has stabilised the two equations may each be set to zero. The first equation gives the following:

$$0 = \gamma_I \Sigma_f \phi / \lambda_I N_{I\,eq}$$

$$N_{I\,eq} = \gamma_I \Sigma_f \phi / \lambda_I$$

The second equation gives, with substitution from the first equation, the following:

$$0 = \gamma_{Xe} \Sigma_f \phi + \lambda_I N_{I\,eq} - \lambda_{Xe} N_{Xe\,eq} - \sigma_{a\,Xe} N_{Xe\,eq} \phi$$

$$N_{Xe\,eq} = (\gamma_{Xe} + \gamma_I)\,\Sigma_f\,\phi\,/\,(\lambda_{Xe} + \sigma_{a\,Xe}\,\phi)$$

The whole process may be modelled by using a water level analogy where water accumulates in two tanks representing Iodine-135 and Xenon-135. Overall inflow is controlled by a valve representing the neutron flux as is the major outflow from the second tank. Outflows representing decay are not controlled but vary according to the level in the tank. If drawn to scale this model allows the process to be easily visualised and any transients can be qualitatively analysed.

XENON REACTIVITY TRANSIENTS

The rise and fall of the concentration of Xenon-135 in a nuclear reactor has a profound effect on its operation. As the Xenon-135 concentration increases neutrons are absorbed and reactivity adjustments must be made to maintain a self sustaining nuclear chain reaction. In order to maintain the neutron multiplication factor k at unity positive reactivity must be added to compensate for the negative reactivity effect of the Xenon-135. Under steady state conditions in a CANDU reactor the Xenon-135 builds up to a concentration which is equivalent of adding 28 mk of negative reactivity. The control systems must be able to compensate for this by adding an equivalent amount of positive reactivity and still maintain an adequate margin for control due to normal purturbations.

Consider a nuclear reactor trip from the full load condition and assume that it remains in the shut down state for an extended period. The Xenon-135 will build up rapidly due to the decay of Iodine-135 and lack of neutron flux to burn up the Xenon-135. Eventually the Xenon-135 will disappear due to its decay. The sudden build up of Xenon-135 can overwhelm the capabilities of the control system due to the addition of so much negative reactivity. At the peak of the Xenon-135 concentration the amount of negative reactivity added in a CANDU reactor is approximately 125 mk. This occurs some 10 hours after the reactor trip. The high concentration of Xenon-135 in the reactor makes it impossible to restart after about 40 minutes in the shut down condition. Some 40 hours must elapse before the Xenon-135 concentration decays to a value which can be overcome by the positive reactivity capabilities of the control system. This is a major constraint in the operation of nuclear reactors. For partial load reductions the transient is not nearly as severe as there is a high enough neutron flux to burn off the excess Xenon-135. In the event of external faults efforts are made to keep the nuclear reactor on a partial load to avoid a forced shut down due to *Xenon poisoning*. This can be done by maintaining a load of about 60% on the reactor and dumping excess steam to the turbine condenser while the fault is corrected on the steam turbine, electrical generator or electrical system. If the fault requires a long time to fix, the

reactor will be tripped and allowed to *poison out*. Naturally any faults on the reactor itself will necessitate an immediate reactor trip.

In the event of an increase in load the opposite will happen. The burnup of Xenon-135 due to the increased neutron flux will effectively create excess positive reactivity in the reactor core. This is not a problem as the control systems are designed to have a big negative reactivity margin.

In large reactors such as the gas cooled reactors and very large CANDU reactors the Xenon-135 concentration may oscillate in different parts of the reactor while the net power output remains steady. The concentration of Xenon-135 may increase on one side while decreasing on the other side creating a *flux tilt*. The danger of this is that, in areas of high local neutron flux, the fuel may be overrated beyond the safe power level. This can result in fuel overheating and damage.

SAMARIUM BUILD-UP

In a nuclear reactor one of the fission products is Neodymium-149 which decays to Promethium-149 by β-particle emission and then to Samarium-149 again by β-particle emission. Samarium-149 is stable and, like Xenon-135, a neutron absorber. The half-life of Promethium-149 is 53 hours so that build up is a relatively slow process. Furthermore Samarium-149 is not as strong a neutron absorber as Xenon-135 and does not have as great an effect. It is also not burnt-up so readily. The overall effect on the reactor is to impose a small permanent negative reactivity effect of about 5 mk. Some 300 hours after a reactor trip and continued shutdown conditions the negative reactivity effect builds up to about 15 mk. The time scale for this is so much longer than that for Xenon-135 that it is of little consequence in normal operation. The permanent build-up of Samarium-149 however does influence the overall reactivity effect of the fuel and must be taken into account when determining reactivity capabilities of new and used fuel or when refuelling the reactor.

FUEL BURN-UP

During operation fissile fuel such as Uranium-235 burns up continuously and eventually reaches a point when it is so depleted that it cannot sustain a nuclear chain reaction. At this time the fuel is replaced with new fuel and the process continued. As the Uranium-235 is burned up, fission products build up in the fuel and some of these are neutron absorbers. On the other hand some Uranium-238 is converted into Plutonium-239 by neutron absorption and two successive β-particle emissions:

$$^{238}U_{92} + {}^{1}n_o = {}^{239}U_{92}$$

$$^{239}U_{92} = {}^{239}Np_{93} + {}^{o}\beta_{-1}$$

$$^{239}Np_{93} = {}^{239}Pu_{94} + {}^{o}\beta_{-1}$$

The depletion of fissile uranium and the creation of plutonium can be represented mathematically.

The destruction of Uranium-235 is given by the following

$$dN_{u\text{-}235}/dt = - \phi \sigma N_{U\text{-}235}$$

$$Nt_{u\text{-}235} = N_{o\ U\text{-}235}\, e^{-\sigma \phi t}$$

The build up of Plutonium-239 is given by the following:

$$dN_{Pu\text{-}239}/dt = \phi \sigma N_{U\text{-}238}$$

The destruction of Plutonium-239 is given by the following if decay is considered to be negligible.

$$dN_{Pu\text{-}239}/dt = - \phi \sigma N_{Pu\text{-}239}$$

The net change of Plutonium-239 is then:

$$dN_{Pu\text{-}239}/dt = \phi \sigma N_{U\text{-}238} - \phi \sigma N_{Pu\text{-}239}$$

$$Nt_{Pu\text{-}239} = N_{eq\ Pu\text{-}239}\,[1 - e^{-\sigma \phi t}]$$

The equilibrium value of Plutonium-239 may be found by setting the rate of change dN/dt equal to zero. The net change of Plutonium-239 can then be determined:

$$N_{eq}\, P_{U\text{-}239} = (\sigma_{U\text{-}238} / \sigma_{Pu\text{-}239})\, N_{U\text{-}238}$$

$$N_{t\ Pu\text{-}239} = (\sigma_{U\text{-}238} / \sigma_{Pu\text{-}239})\, N_{U\text{-}238}\,[1 - e^{-\sigma \phi t}]$$

The Plutonium-239 builds up quite rapidly at first. Some of it absorbs neutrons in two stages to become Plutonium-241, another fissile fuel, but the concentration of that never reaches

significant proportions within the normal fuel life time. Initially the build up of Plutonium-239 actually causes an increase in the reactivity of the core. This is due to its higher fission cross section as well as the greater number of neutrons produced per fission when compared with Uranium-235. Ultimately the depletion of Uranium-235 and the build up of fission products in the reactor causes a decrease in the reactivity of the core. In a CANDU reactor fuel is removed from the reactor when roughly one half to two thirds of the Uranium-235 has been consumed.

The effect of fuel burnup on the reactivity of the reactor is primarily through factor η the number of neutrons emitted per neutron absorbed.

$$\eta = \nu \Sigma_{f\ fuel} / \Sigma_{a\ fuel}$$

As the fuel is burned up $\Sigma_{f\ fuel}$ decreases and as fission products accumulate $\Sigma_{a\ fuel}$ increases. The overall effect is that η decreases with time. The effects of Samarium-149 and Plutonium-239 can however be seen. At the very beginning there is a drop in η due to the build up of Samarium-149 a neutron absorber. Somewhat later but still early in the fuel cycle the value of η increases and reaches a peak due to the formation of Plutonium-239.-

APPROACH TO CRITICALITY

When a nuclear reactor is started up from the shut down condition the neutron flux level has to be increased by several orders of magnitude. In the shut down condition the neutron flux is determined by the intensity of the neutron sources and the subcritical multiplication factor. The value of the neutron multiplication factor k is substantially less than unity and the reactor is subcritical. In starting up the reactor the value of k is adjusted to bring it to unity making the reactor critical and capable of sustaining a continuous fission chain reaction. In so doing the subcritical multiplication factor increases the neutron flux level since Δk becomes smaller and smaller.

$$S_\infty = S_o / (1 - k)$$

$$S_\infty = - S_o / \Delta k$$

Two interesting aspects arise from this. Firstly, as Δk approaches zero, the measured neutron flux S_∞ theoretically goes to infinity. This is obviously not practical since critical conditions must be established at a measurable neutron flux level. The real situation is that Δk can only be measured and controlled to a certain degree of accuracy. Once this limit has been reached

slight purturbations cause control system responses that mask the actual of Δk and the reactor can be considered to be oscillating very slightly above and below criticality. Under these conditions the reactor is considered to be critical with the average value of k equal to unity. Secondly, since the value of the source neutron flux S_o can be very small the nuclear reactor can become critical at any power level once the slightly oscillating conditions just described have been achieved. The net result is that, during reactor startup, criticality can be reached at any power level and there is no direct indication of when criticality is achieved.

A further compliication is that, even though a nuclear reactor may be shut down by inserting a known amount of negative reactivity the reactivity within the reactor may change with time due to the build up or decay of fission products. This means that one cannot simply restart a reactor by inserting an amount of positive reactivity equal to the amount of negative reactivity inserted to shut it down.

Approach to criticality is thus a delicate manoeuvre as if an approach to a moving and invisible target was being made. At every step therefore an assessment has to made as to how far from criticality the reactor actually is. Each step must also be small enough so that if the point of criticality is overshot the reactor does not become prompt critical when in the supercritical range. This means that Δk must not be more than β the delayed neutron fraction which is approximately equal to 0.007 for uranium fuel.

It is obvious that a reactor cannot be brought to the critical condition in very small steps so that Δk never approaches +0.007 from a shut down state where Δk may be somewhere around -0.90. A suitable procedure must therefore be devised and rigorously applied to ensure safety of the reactor during these manoeuvres.

NUCLEAR REACTOR STARTUP

When starting up a nuclear reactor neutron counters are required to measure the neutron density or neutron flux in the reactor. These instruments are usually situated on the side of the reactor but other removable ones may be inserted into the core of the reactor when raising power from very low levels when the neutron flux cannot be detected by the normal instrumentation. The count rate on the instruments is a measure of the reactor power.

While Δk has a large negative value a change in reactivity will be reflected quickly on the instruments as the fission chain reactions initiated by the source neutrons die away rapidly since the value of k is far below unity. Following an insertion of some positive reactivity the reactor power will rise quickly to a new equilibrium value. When however Δk has a small

negative value and k is very close to unity the reactor will respond very much more slowly. In these circumstances the fission chain reactions initiated by the source neutrons persist for many generations. Following insertion of some positive reactivity the reactor power will rise slowly but more significantly and take much long to settle at its new equilibrium value. When the reactor becomes critical due to the insertion of additional positive reactivity the reactor power will rise steadily and continuously without levelling off at an equilibrium value. Since the reactor will be very slightly supercritical the power will in fact rise exponentially. If the power is plotted on a chart recorder as was the case for the very first nuclear reactor at Chicago University it is evident from the plot when criticality is achieved. Generally such a crude mechanism of monitoring the approach to criticality is not suitable for commercial reactors. By analysing carefully what happens during the approach to criticality it is possible to derive a more positive and safer method.

Consider an example where a reactor has been shut down for a few days with -100 mk ($\Delta k = -0.1$) of reactivity. Assume that the predominant source of neutrons is by photoneutron action. If this photoneutron power is 0.001% (0.00001) of full reactor power then the measured reactor power is:

$$P_\infty = -P_0 / \Delta k = 0.0001 \ (0.01\%)$$

If it is assumed that the measured reactor power P_∞ is doubled at each step then various parameters such as k and Δk can be calculated to show their changes. The count rate can be assumed to be proportional to the measured neutron flux S_∞ and the inverse count rate determined. The results of this power doubling at each step can be summarised as follows:

- The value of k progresses half way to unity at each step.
- The reactivity added at each step to double the power deceases.
- The source power P_0 becomes an insignificant fraction of the measured power P_∞.
- The inverse of the count rate decreases towards zero.

These changes may be plotted graphically. A plot of power versus steps to critical is linear. This is not useful since the reactor can go critical at any power. A plot of k versus steps to critical shows an asymtotic approach to the critical condition where k is equal to unity. This shows clearly that reactivity should be added in smaller and smaller steps as the reactor

approaches criticality since small reactivity changes near criticality will effect large changes in reactor power. It is as if the reactor becomes ultra sensitive as criticality is approached. The most useful plot however is inverse of count rate versus reactivity added. This shows that criticality is achieved when the inverse of count rate becomes zero. The plot is linear enabling the amount of reactivity to be added to achieve criticality to be easily determined.

In starting up a reactor with an unknown amount of negative reactivity, that is, an unknown shut down value for k a plot of inverse count rate versus reactivity added is made. Reactivity is gradually added until the power has doubled. When far from critical conditions the response is quick and the reactivity required to achieve this doubling of power can be determined easily. To reach critical conditions would now require an equal amount of reactivity to be added since any doubling of power requires the addition of half the reactivity that would be required to go directly to critical conditions. To be conservative however a lesser amount of reactivity is added and a new point plotted. Should the curve not be linear this would avoid a possible excursion into the supercritical region. With each measurement and plotted point the exact point of criticality can be more accurately predicted. In this way a nuclear reactor can be safely started up.

It should be noted that reactivity can be related to the adjustment of various control devices. Control rods for example have a certain reactivity worth as does a certain concentration in the moderator of a soluble neutron absorber such as boron. The inverse count rate can thus be plotted directly against control rod position or boron concentration. Non-linearities in the count rate curve arise due to non-linear neutron absorption characteristics which is why several predictions of the point of criticality are desirable.

The numerical examples of approach to critical show the effect of not rounding off the values of k. It appears that the reactor will never go critical if the calculations are done with a high enough degree of accuracy. In practice the control and instrumentation systems cannot match this degree of accuracy and the reactor becomes critical once it enters the dead band of the control system.

The effect of an extended shut down period when the source neutron power has decayed to a value of only 0.000 01% is also shown. The effect of this is that the reactor goes critical at a correspondingly low value of measured reactor power.

The numerical examples also show the extreme range required in the measuring instrumentation. Neutron flux ranges over some 15 decades while most instruments can only cover about 5 decades. There are thus instruments in several positions where the neutron

flux is different so that, as the power changes, different instruments can be used to measure the power. Most instruments are duplicated or triplicated to ensure sufficient redundancy. Often a reactor signal is validated only if two out of three instruments give consistent readings.

Reactor Power

Neutron Flux

ϕ = nv (neutrons/cm^{2}s)

n = Neutron density (neutrons/cm^3)

v = Neutron velocity (cm/s)

Macroscopic cross-section

Σ_f = $N\sigma_f$ (cm^{-1})

N = Nuclei density (nuclei/cm^3)

σ_f = Microscopic cross-section (cm^2)

Reactor power

$P = \Sigma_f \phi V E_f \quad \frac{1}{cm} \times \frac{1}{cm^3 s} \times \frac{cm^3}{1} \times \frac{J}{1}$

V = Volume of reactor (cm^3)

E_f = Energy release per fission (J)

OH 8.1

REACTOR KINETICS

INITIAL NEUTRON DENSITY = n

NEUTRON DENSITY IN NEXT GENERATION = kn

CHANGE IN NEUTRON DENSITY

$$\Delta n = kn - n$$

TIME FOR CHANGE IS NEUTRON LIFETIME = l

RATE OF CHANGE IS CHANGE PER UNIT TIME

$$\frac{\Delta n}{\Delta t} = \frac{kn - n}{l}$$

$$\frac{dn}{dt} = \frac{n}{l}(k-1) = \frac{n}{l}\Delta k$$

$$\frac{dn}{n} = \frac{\Delta k}{l} dt$$

$$\int_0^t \frac{dn}{n} = \int_0^t \frac{\Delta k}{l} dt$$

$$\left[\ln n\right]_0^t = \frac{\Delta k}{l}\left[t\right]_0^t$$

$$\ln n_t - \ln n_0 = \frac{\Delta k}{l} t$$

$$\ln_e \frac{n_t}{n_0} = \frac{\Delta k}{l} t$$

$$\frac{n_t}{n_0} = e^{\frac{\Delta k}{l} t}$$

$$n_t = n_0 e^{\frac{\Delta k}{l} t}$$

$$P_t = P_0 e^{\frac{\Delta k}{l} t}$$

Reactor Period

Reactor period τ is the time taken for the power to increase by a factor of e

$$P_t = P_0 e^{\frac{\Delta k}{l}\tau}$$

Thus after one reactor period

$$t = \tau$$

and $P_t = eP_0$

Thus by substitution

$$e = e^{\frac{\Delta k}{l}\tau}$$

$$l = \frac{\Delta k}{l}\tau$$

$$\tau = \frac{l}{\Delta k}$$

OH 8.2

Prompt and Delayed Neutrons in Multiplication Process

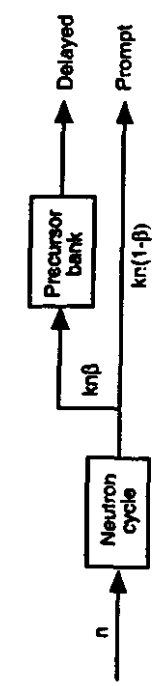


Fig. 8.2

EFFECT OF DELAYED NEUTRONS

PREVIOUS EQUATION

$$\frac{\Delta n}{\Delta t} = \frac{kn - n}{l}$$

$$\frac{dn}{dt} = \frac{kn}{l} - \frac{n}{l}$$

A FRACTION β OF NEUTRONS ARE DELAYED

$$\frac{dn}{dt} = \frac{kn}{l}(1-\beta) - \frac{n}{l}$$

DELAYED NEUTRONS APPEAR BY DECAY OF PRECURSORS

$$\left(\frac{dn}{dt}\right)_{\text{DELAYED NEUTRONS}} = \lambda C$$

$$\frac{dn}{dt} = \frac{kn}{l}(1-\beta) + \lambda C - \frac{n}{l}$$

THE PRECURSOR BANK EQUATION IS

$$\frac{dC}{dt} = \frac{kn\beta}{l} - \lambda C$$

FOR EQUILIBRIUM VALUE OF PRECURSOR BANK

AND FOR $\Delta k < \beta$

$$P_t = P_0\left[\left(\frac{\beta}{\beta-\Delta k}\right)e^{\left(\frac{\lambda\Delta k}{\beta-\Delta k}\right)t} - \left(\frac{\Delta k}{\beta-\Delta k}\right)e^{-\left(\frac{\beta-\Delta k}{l}\right)t}\right]$$

AFTER A SHORT PERIOD OF TIME

$$P_t = P_0\left(\frac{\beta}{\beta-\Delta k}\right)e^{\left(\frac{\lambda\Delta k}{\beta-\Delta k}\right)t}$$

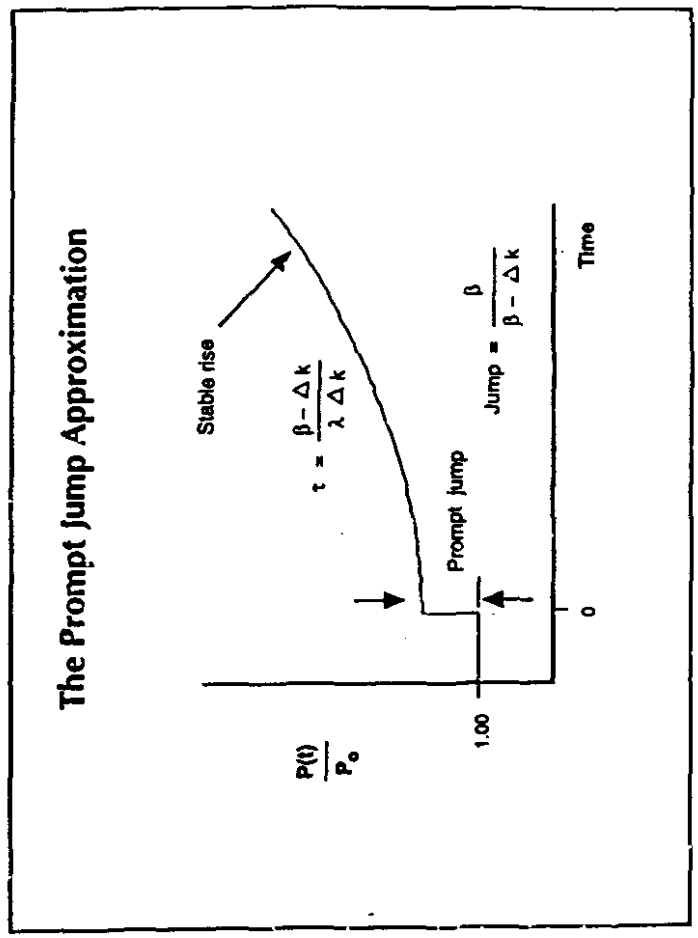


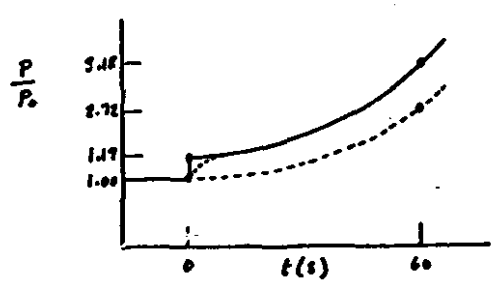
Fig. 8.5

REACTIVITY EQUATION

$$P_t = P_0\left[\frac{\beta}{\beta-\Delta k}e^{\frac{\lambda\Delta k}{\beta-\Delta k}t} - \frac{\Delta k}{\beta-\Delta k}e^{-\frac{\beta-\Delta k}{l}t}\right]$$

FOR $\Delta k = 0.001$, $\beta = 0.007$, $\lambda = 0.1\ s^{-1}$, $l = 0.001\ s$

$$P_t = P_0\left[1.17\,e^{0.0167t} - 0.17\,e^{-1.0t}\right]$$



PROMPT JUMP $P_J = \frac{\beta}{\beta-\Delta k} = 1.17$

REACTOR PERIOD $\tau = \frac{\beta-\Delta k}{\lambda\Delta k} = 60\ s$

AFTER 1 PERIOD, THAT IS, AT $t = 60\ s$

$$P_t = P_0\left[1.17e^{1} - 0.17e^{-60}\right] = P_0\,1.17 \times 2.718 = P_0\,3.18$$

SIMPLE EQUATION (WITHOUT PROMPT JUMP)

$$P_t = P_0 e^{t/\tau} = P_0 e^{1} = P_0\,2.718$$

DEFINITION OF PERIOD EXCLUDES THE PROMPT JUMP

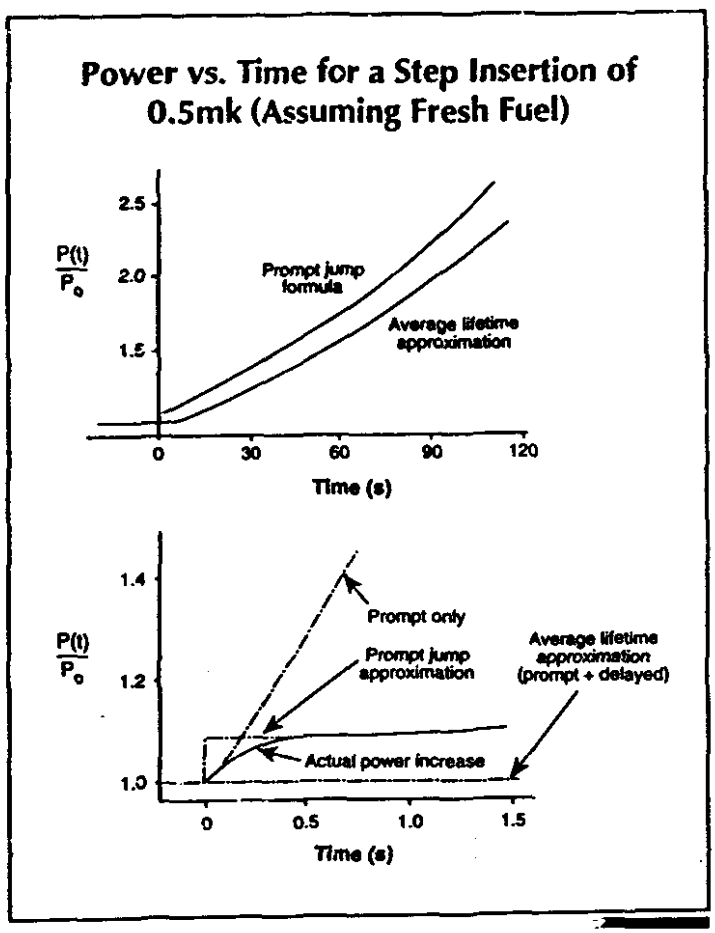
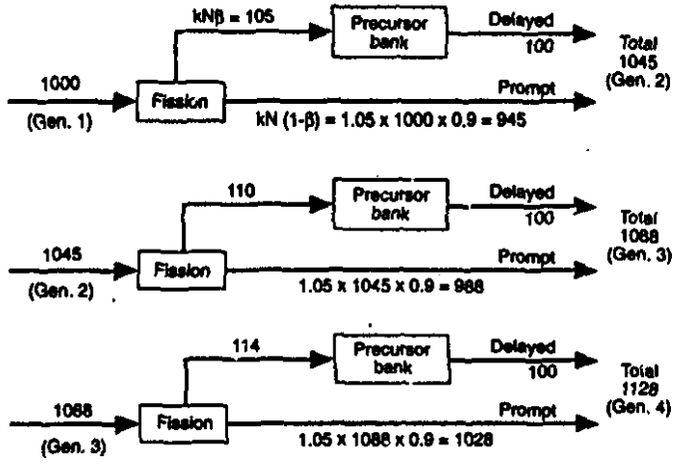


Fig. 8.6

Initial Response to a Change in Reactivity



Delayed Neutron Precursor Groups for U-235 Fission

Precursor group	Half life (seconds)	Relative yield (%)
1	55	4
2	22	21
3	6	19
4	2.3	40
5	0.5	13
6	0.2	3

Situation After Many Generations

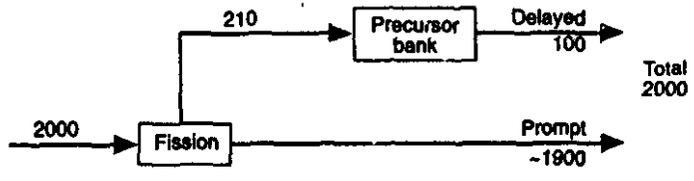
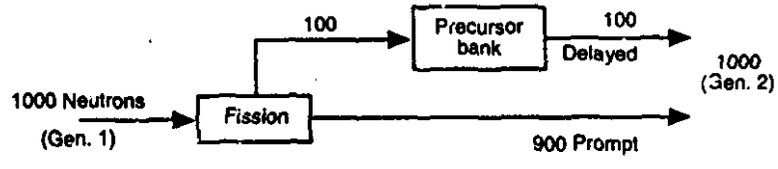


Fig. 8.4

Steady State



OH 8.3

Negative Reactivity

Power due to prompt drop

$$P_1 = P_o \frac{\beta}{\beta - \Delta k}$$

where Δk is negative

Power following prompt drop will decrease with period given by:

$$\tau = \frac{\beta - \Delta k}{\lambda \Delta k}$$

if $\Delta k >> \beta$

$$\tau = \frac{+\Delta k}{-\lambda \Delta k}$$

$$= \frac{1}{-\lambda}$$

Thus

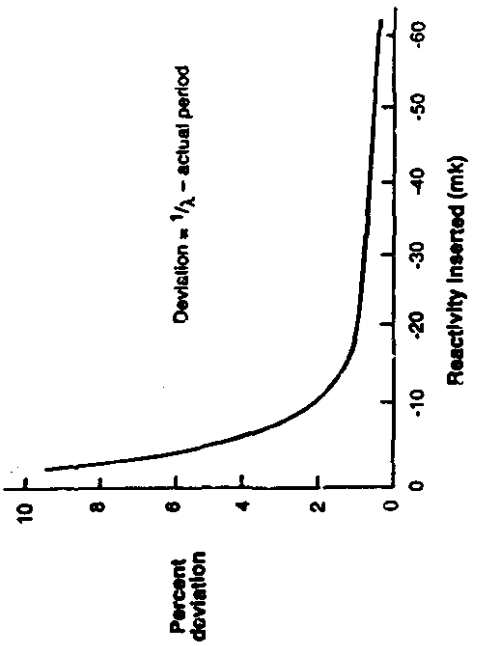
$$P_t = P_1 e^{t/\tau}$$

$$= P_1 e^{-\lambda t}$$

Where λ is decay constant of delayed neutron precursers

OH 8.4

Accuracy of Approximation $\tau \sim -1/\lambda$ (Section 8.7) For Large Reactivity Insertion

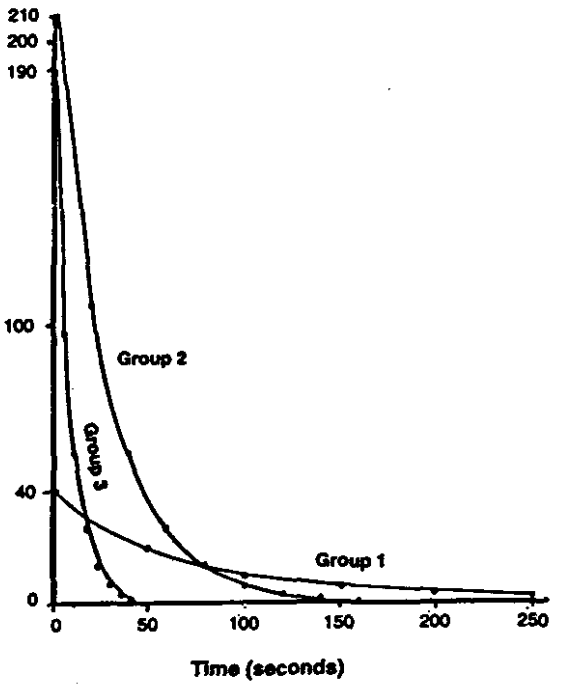


OH 8.7

Delayed Neutron Precursor Groups for U-235 Fission

Precurser group	Half life (seconds)	Relative yield (%)
1	55	4
2	22	21
3	6	19
4	2.2	40
5	0.5	13
6	0.18	3

Decay of Delayed Neutron Precursors (1000 Initially)



OH 8.6

Neutron Sources (After Shutdown)

Spontaneous fission

U-235	0.3 DIS/S per Kg
U-238	6.9 DIS/S per Kg

Decay of delayed neutron precursers

Delayed neutron fraction	0.0070
Relative yield of group 1	0.04
Half life of group 1	55 s

Photo-neutron emission

$^{9}Be_4 + {}^{0}\gamma_0 = {}^{8}Be_4 + {}^{1}n_0 \qquad \gamma > 1.6$ MeV

$^{9}Be_4 + {}^{0}\gamma_0 = {}^{4}He_2 + {}^{4}He_2 + {}^{1}n_0$

$^{2}H_1 + {}^{0}\gamma_0 = {}^{1}H_1 + {}^{1}n_0 \qquad \gamma > 2.2$ MeV

γ energy required > 2.2 MeV (CANDU)

Half life of fission product with γ energy > 2.2 MeV is 15 days

OH 9.1

Source Multiplication

Source neutrons : S_0 per generation

Neutron Multiplication factor : k

Neutrons after 1 generation : S_0

Neutrons after 2 generations : $S_0k + S_0$

Neutrons after 3 generations : $S_0k^2 + S_0k + S_0$

Neutrons after ∞ generations :

$$S_\infty = S_0 + S_0k + S_0k^2 + S_0k3 + S_0k^4 + \ldots\ldots$$
$$= S_0 (1 + k + k^2 + k^3 + k^4 + \ldots\ldots)$$
$$= S_0 \frac{1}{1-k}$$
$$= \frac{S_0}{1-k}$$

$$\Delta k = k-1$$

$$S_\infty = -\frac{S_0}{\Delta k}$$

$$P_\infty = \frac{P_0}{1-k}$$

Subcritical multiplication factor $= \frac{1}{1-k}$

In a subcritical reactor source neutrons cause a substantial neutron population

OH 9.2

Rate Log Power

Power change

$$P = P_0 e^{t/\tau}$$

Power change over a given time interval

$$P = P_0 e^{\Delta t/\tau}$$

$$\text{Ln } P = \text{Ln } P_0 + \text{Ln } e^{\Delta t/\tau}$$

$$= \text{Ln } P_0 + \frac{\Delta t}{\tau}$$

$$\text{Ln } P - \text{Ln } P_0 = \frac{\Delta t}{\tau}$$

$$\Delta[\text{Ln } P] = \frac{\Delta t}{\tau}$$

$$\Delta[\text{Ln } P]/\Delta t = \frac{1}{\tau}$$

Rate log = Change in log power per unit time

$$= \frac{1}{\text{PERIOD}}$$

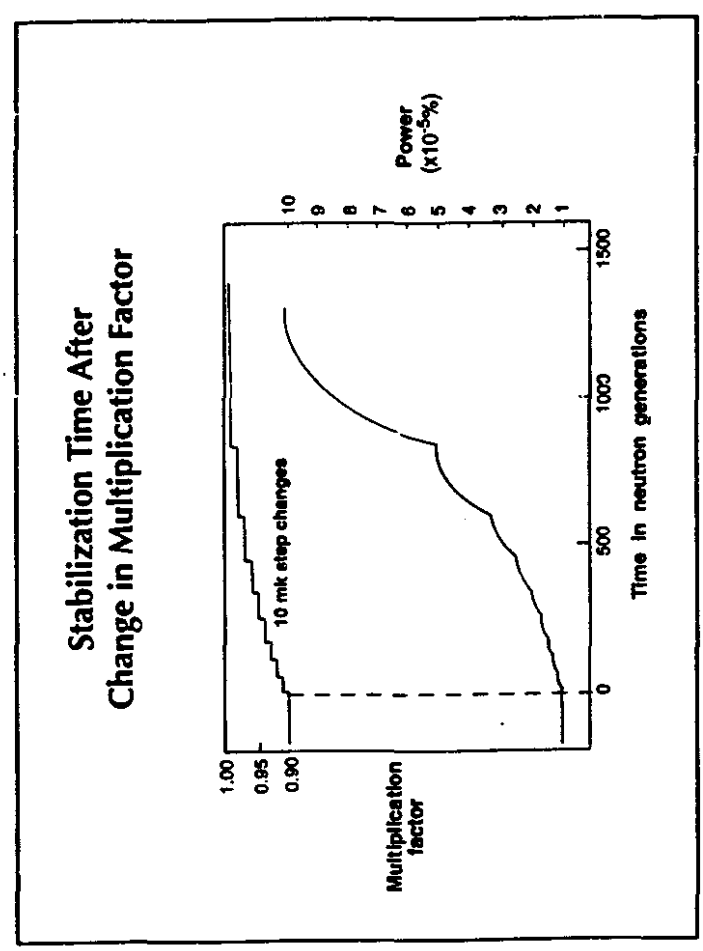
Note conversion from LN_e to LOG_{10}

$$LN_eP = 2.3026\ LOG_{10}P$$

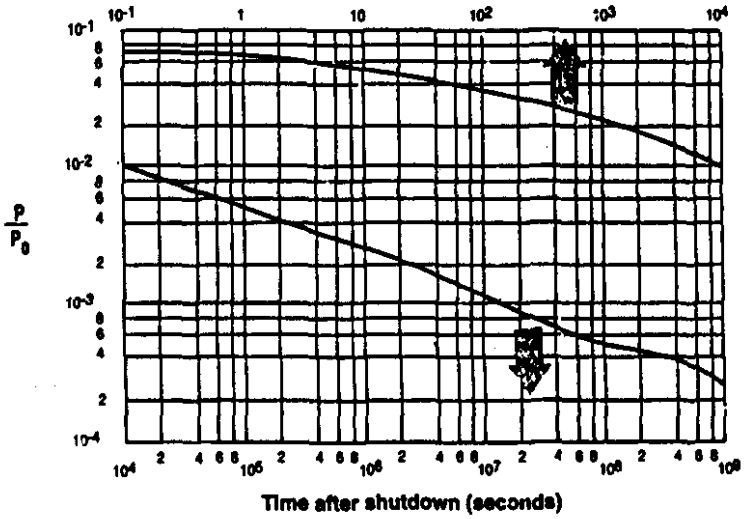
$$LOG_{10}P = LN_eP / 2.3026$$

OH 10.2

Fig. 9.1

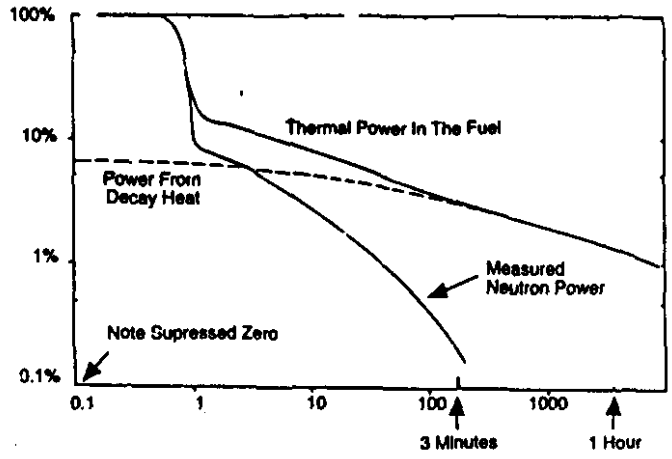


Decay Heat Rundown



OH 10.1

Decay of Neutron and Thermal Power After Shutdown



Details of Decay of Neutron Power After Shutdown (on two time scales)

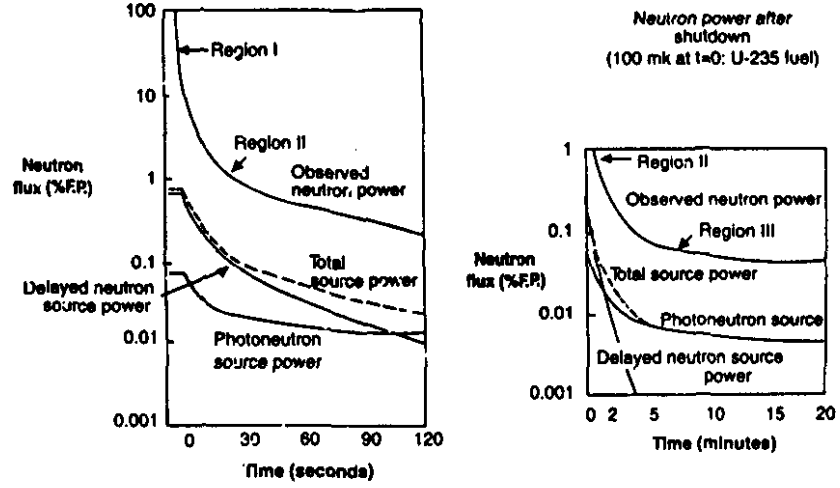


Fig. 10.2

Xenon Build-up

Build-up of I-135 = $\gamma_I \Sigma_f \phi$

Decay of I-135 = $\lambda_I N_I$

Rate of change of I-135

$$\frac{dN_I}{dt} = \gamma_I \Sigma_f \phi - \lambda_I N_I$$

Build-up of Xe-135 = $\gamma_{Xe} \Sigma_f \phi$

Production of Xe-135 = $\lambda_I N_I$

Decay of Xe-135 = $\lambda_{Xe} N_{Xe}$

Burn-up of Xe-135 = $\sigma_{axe} N_{Xe} \phi$

Rate of change of Xe-135

$$\frac{dN_{Xe}}{dt} = \gamma_{Xe} \Sigma_f \phi + \lambda_I N_I - \lambda_{Xe} N_{Xe} - \sigma_{axe} N_{Xe} \phi$$

At Equilibrium: $\frac{dN_I}{dt} = 0$, $\frac{dN_{Xe}}{dt} = 0$,

$$0 = \gamma_I \Sigma_f \phi - \lambda_I N_{Ieq}$$

$$N_{Ieq} = \frac{\gamma_I \Sigma_f \phi}{\lambda_I}$$

$$0 = \gamma_{Xe} \Sigma_f \phi + \lambda_I N_{Ieq} - \lambda_{Xe} N_{Xe\,eq} - \sigma_{axe} N_{Xe\,eq} \phi$$

$$N_{Xe\,eq}(\lambda_{Xe} + \sigma_{axe} \phi) = \gamma_{Xe} \Sigma_f \phi + \frac{\lambda_I \gamma_I \Sigma_f \phi}{\lambda_I}$$

$$N_{Xe\,eq} = (\gamma_{Xe} + \gamma_I)\, \Sigma_f \phi / (\lambda_{Xe} + \sigma_{axe} \phi)$$

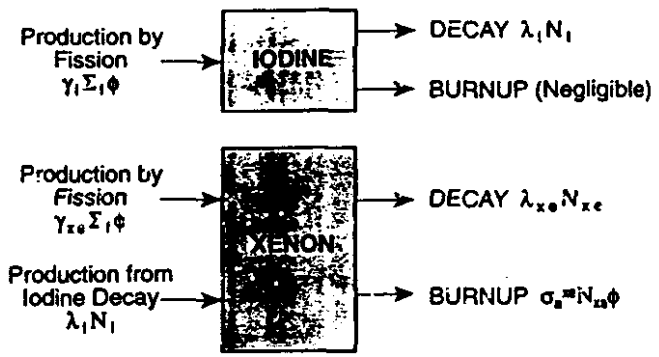
$$\approx (\gamma_{Xe} + \gamma_I)\, \Sigma_f \phi / (\sigma_{axe} \phi)$$

OH 11.1

Production and Loss Rates of Iodine and Xenon

$$^{135}I \xrightarrow[6.7h]{\beta} {}^{135}Xe \xrightarrow[9.2h]{\beta} {}^{135}Cs$$

$\gamma_I = 0.063$ $\quad \gamma_{Xe} = 0.003$ $\quad \sigma_{Xe} = 3.5 \times 10^6$ b



$$\frac{d}{dt}(N_I) = \gamma_I \Sigma_f \phi - \lambda_I N_I$$

$$\frac{d}{dt}(N_{Xe}) = [\gamma_{Xe} \Sigma_f \phi + \lambda_I N_I] - [\lambda_{Xe} N_{Xe} + \sigma_a^{Xe} N_{Xe} \phi]$$

5% 95% 10% 90%

Fig. 11.1

"Water Level" Analogy of Xenon Formation

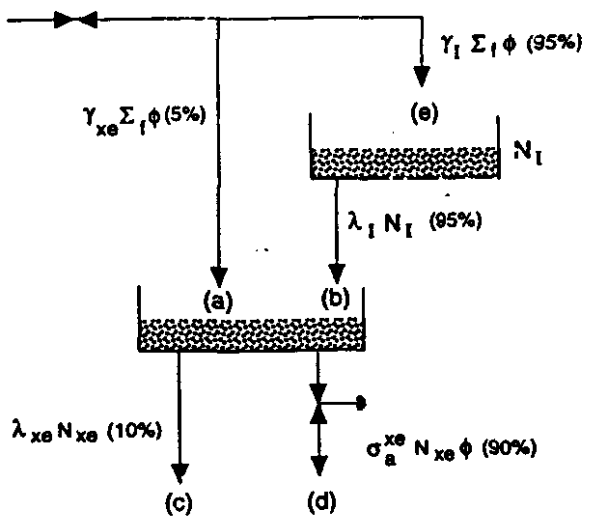
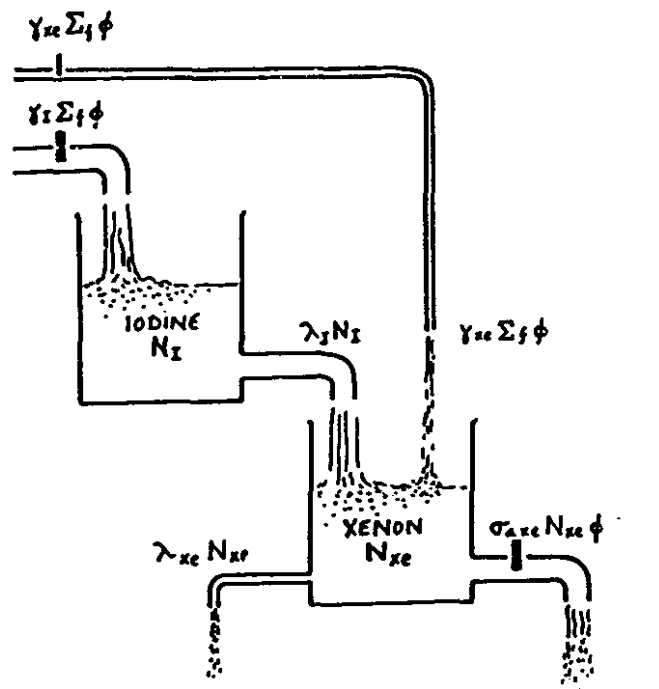


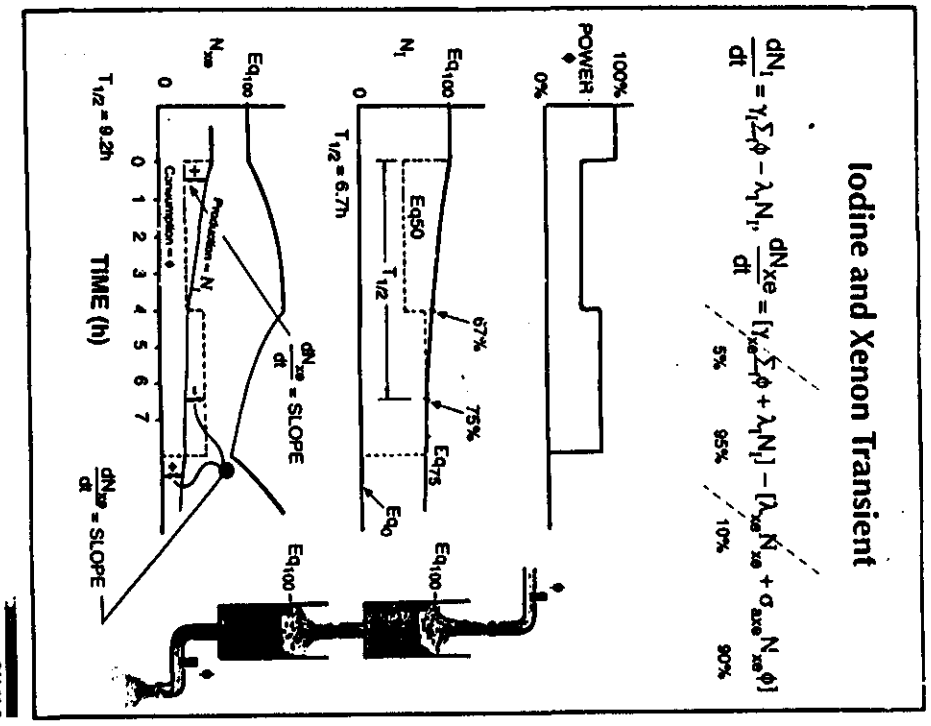
Fig. 11.6

IODINE & XENON



$$\frac{dN_I}{dt} = \gamma_I \Sigma_f \phi - \lambda_I N_I$$

$$\frac{dN_{Xe}}{dt} = [\gamma_{Xe} \Sigma_f \phi + \lambda_I N_I] - [\lambda_{Xe} N_{Xe} + \sigma_{a\,Xe} N_{Xe} \phi]$$



OH 11-2

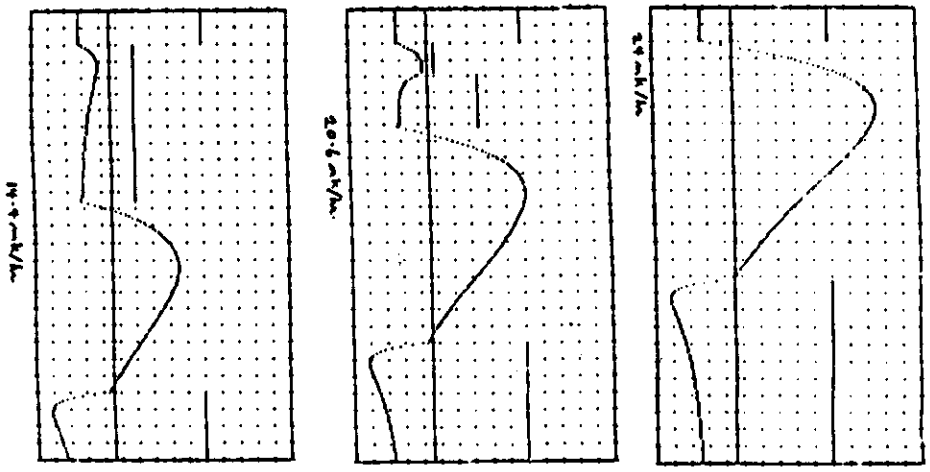


TABLE 7.5
Fission product yields (atoms per fission) from thermal fission*

Isotope	^{233}U	^{235}U	^{239}Pu
^{135}I	0.0475	0.0639	0.0604
^{135}Xe	0.0107	0.00237	0.0105
^{149}Pm	0.00795	0.01071	0.0121

*From M. E. Meek and B. F. Rider, "Compilation of Fission Product Yields," General Electric Company Report NEDO-12154, 1972.

TABLE 7.6
Decay constants for fission product poisoning calculations

Isotope	λ, sec^{-1}	λ, hr^{-1}
^{135}I	2.87×10^{-5}	0.1035
^{135}Xe	2.09×10^{-5}	0.0753
^{149}Pm	3.63×10^{-6}	0.0131

Build-up of I-135 Towards Equilibrium Value

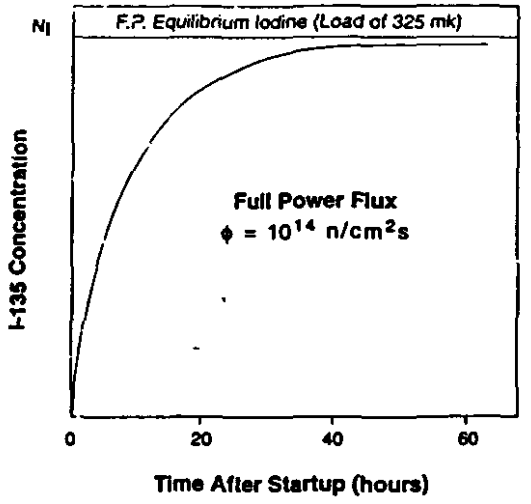


Fig. 11.2

Build-up of Xe-135 Towards Equilibrium Value

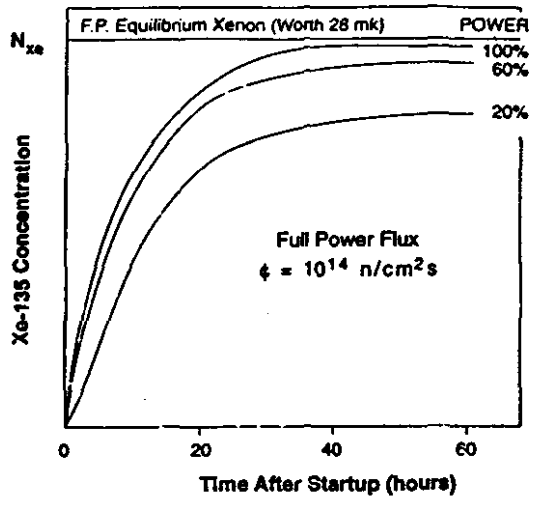
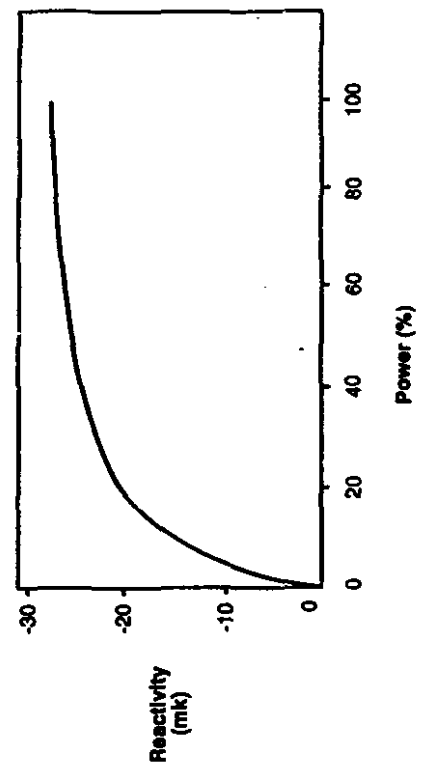


Fig. 11.3

Xenon Load as a Function of Reactor Power



Xenon Reactivity Transient For Point Lepreau CANDU 600

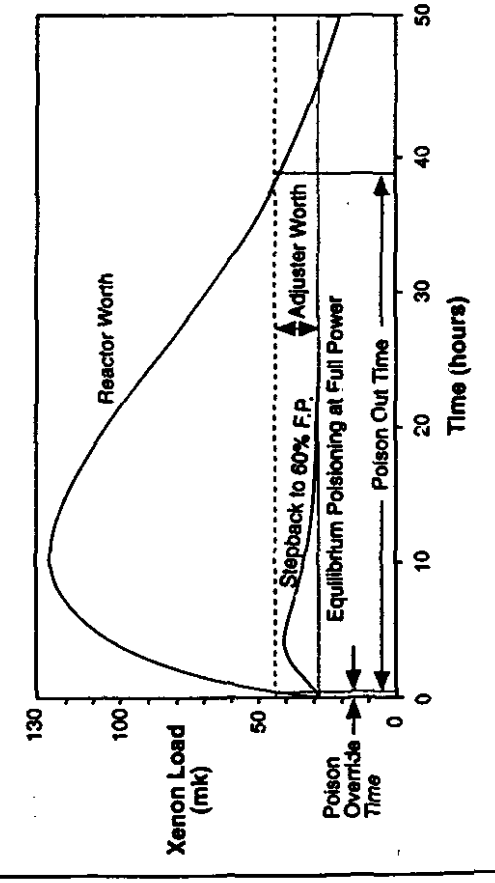


Fig. 11.5

Xenon transients following step reductions in power from an initial power of 200MWe. Xenon is assumed to be in steady state originally; fuel assumed to be at equilibrium irradiation

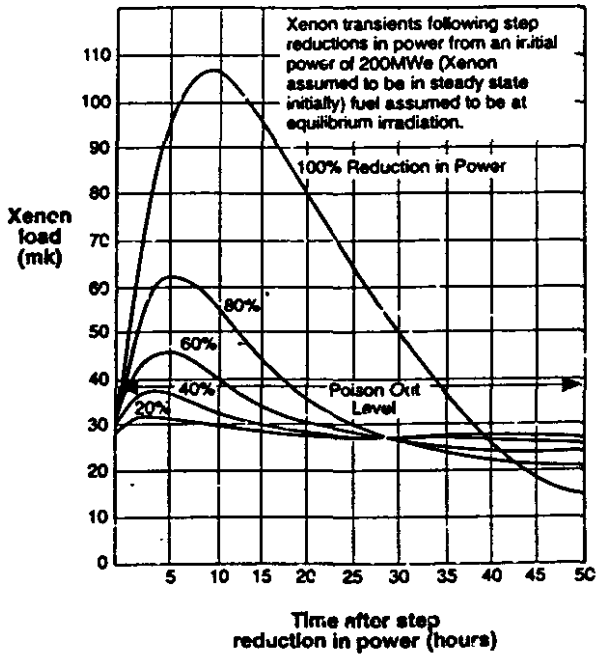


Fig. 11.7

Variation in Xenon Load Following Increase in Power From the Various Initial Levels (% of Full Power) Indicated

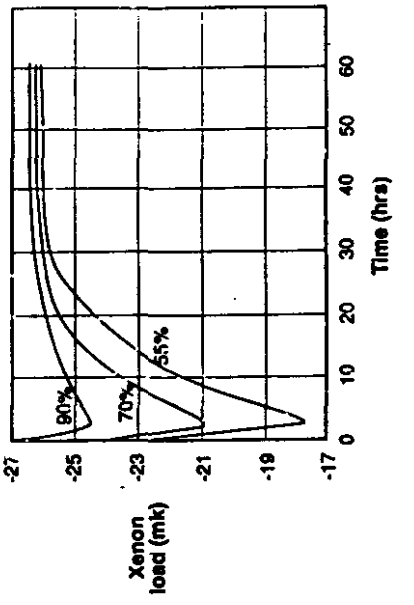


Fig. 11.9

Samarium Build-up

NEODYMIUM ⟶ PROMETHIUM ⟶ SAMARIUM

$$Nd-149 \xrightarrow[1.7h]{\beta} Pm-149 \xrightarrow[53h]{\beta} Sm-149$$

Build-up of Pm -149 = $\gamma_{Pm}\Sigma_f\phi$

Decay of Pm -149 = $\lambda_{Pm}N_{Pm}$

Rate of change of Pm -149 $\frac{dN_{Pm}}{dt} = \gamma_{Pm}\Sigma_f\phi - \lambda_{Pm}N_{Pm}$

Production of Sm-149 = $\lambda_{Pm}N_{Pm}$

Burn-up of Sm -149 = $\sigma_{asm}N_{Sm}\phi$

Rate of change of Sm-149 $\frac{dN_{Sm}}{dt} = \lambda_{Pm}N_{Pm} - \sigma_{aSm}N_{Sm}\phi$

At Equilibrium: $\frac{dN_{Pm}}{dt} = 0, \frac{dN_{Sm}}{dt} = 0$.

$0 = \gamma_{Pm}\Sigma_f\phi - \lambda_{Pm}N_{Pm\,eq}$

$N_{Pm\,eq} = \frac{\gamma_{Pm}\Sigma_f\phi}{\lambda_{Pm}}$

$0 = \lambda_{Pm}N_{Pm\,eq} - \sigma_{aSm}N_{Sm\,eq}\phi$

$N_{Sm\,eq} = \frac{\lambda_{Pm}N_{Pm\,eq}}{\sigma_{aSm}\phi}$

$= \frac{\lambda_{Pm}\gamma_{Pm}\Sigma_f\phi}{\sigma_{aSm}\phi\lambda_{Pm}} = \frac{\gamma_{Pm}\Sigma_f}{\sigma_{aSm}}$

OH 11.3

Increase In Samarium Load after Shutdown

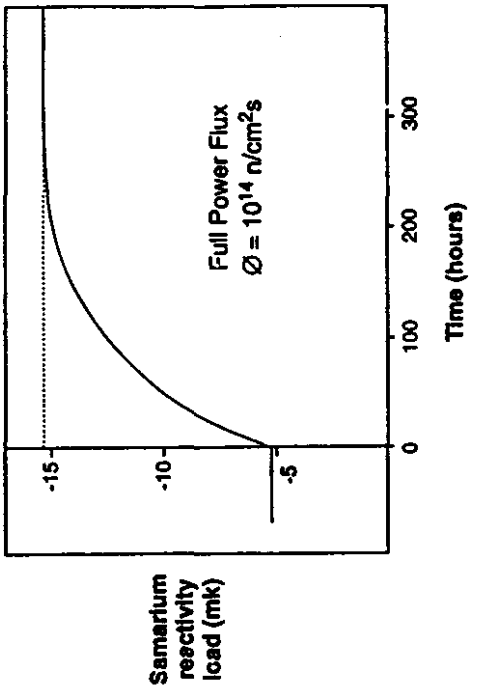


Fig. 11.10

FUEL BURNUP

DESTRUCTION OF U-235

$$\frac{dN}{dt} = -\phi \sigma N_{u\text{-}235}$$

SUBSTITUTE $\frac{dN}{dt} = -\lambda N_{u\text{-}235}$

SOLVE $N_t = N_0 e^{-\lambda t}$

SUBSTITUTE $N_t = N_0 e^{-\sigma \phi t}$

BUILD-UP OF Pu-239

$$\frac{dN}{dt} = \phi \Sigma_{u\text{-}238}$$

$$\frac{dN}{dt} = \phi \sigma N_{u\text{-}238}$$

DESTRUCTION OF Pu-239

$$\frac{dN}{dt} = -\phi \sigma N_{Pu\text{-}239} - \lambda N_{Pu\text{-}239}$$

DECAY IS NEGLIGABLE

$$\frac{dN}{dt} = -\phi \sigma N_{Pu\text{-}239}$$

NET CHANGE OF Pu-239

$$\frac{dN}{dt} = \phi \sigma N_{u\text{-}238} - \phi \sigma N_{Pu\text{-}239}$$

$$\frac{dN}{dt} = \phi \sigma N_{u\text{-}238} - \phi \sigma N_{Pu\text{-}239}$$

SUBSTITUTE $\frac{dN}{dt} = \phi \Sigma_{u\text{-}238} - \lambda N_{Pu\text{-}239}$

SOLVE $N_t = N_{eq\,Pu\text{-}239} \left[1 - e^{-\lambda t}\right]$

SUBSTITUTE $N_t = N_{eq\,Pu\text{-}239} \left[1 - e^{-\sigma \phi t}\right]$

SOLVE FOR N_{eq} WHEN AT EQUILIBRIUM

$$\frac{dN}{dt} = 0$$

$$\phi \sigma N_{u\text{-}238} - \phi \sigma N_{eq\,Pu\text{-}239} = 0$$

$$\sigma N_{u\text{-}238} = \sigma N_{eq\,Pu\text{-}239}$$

$$N_{eq\,Pu\text{-}239} = \sigma_{u\text{-}238} N_{u\text{-}238} / \sigma_{Pu\text{-}239}$$

$$N_{eq\,Pu\text{-}239} = (\sigma_{u\text{-}238} / \sigma_{Pu\text{-}239}) N_{u\text{-}238}$$

SUBSTITUTE BACK INTO SOLUTION

$$N_t = (\sigma_{u\text{-}238} / \sigma_{Pu\text{-}239}) N_{u\text{-}238} \left[1 - e^{-\sigma \phi t}\right]$$

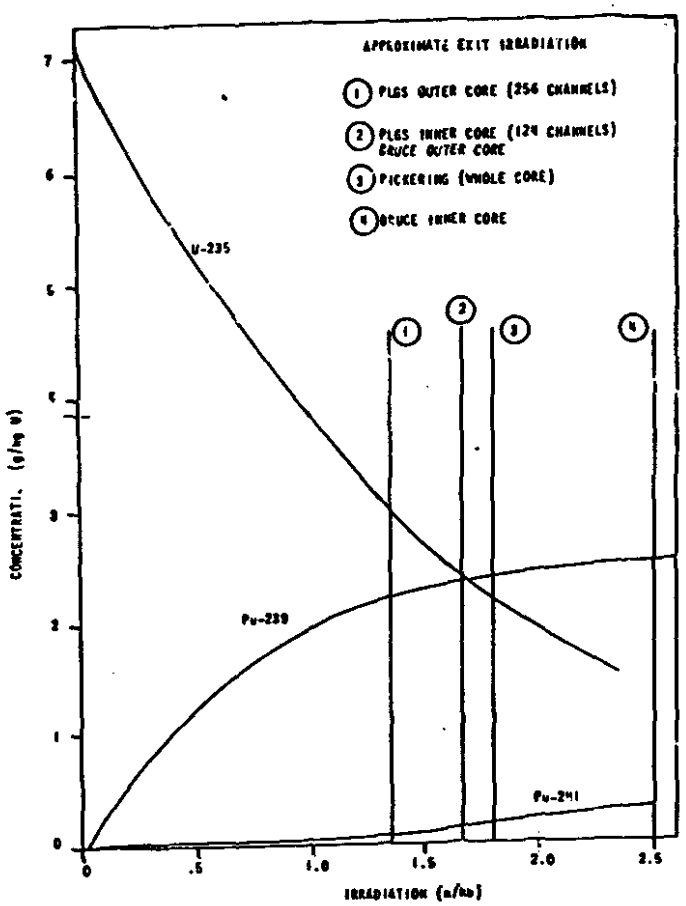


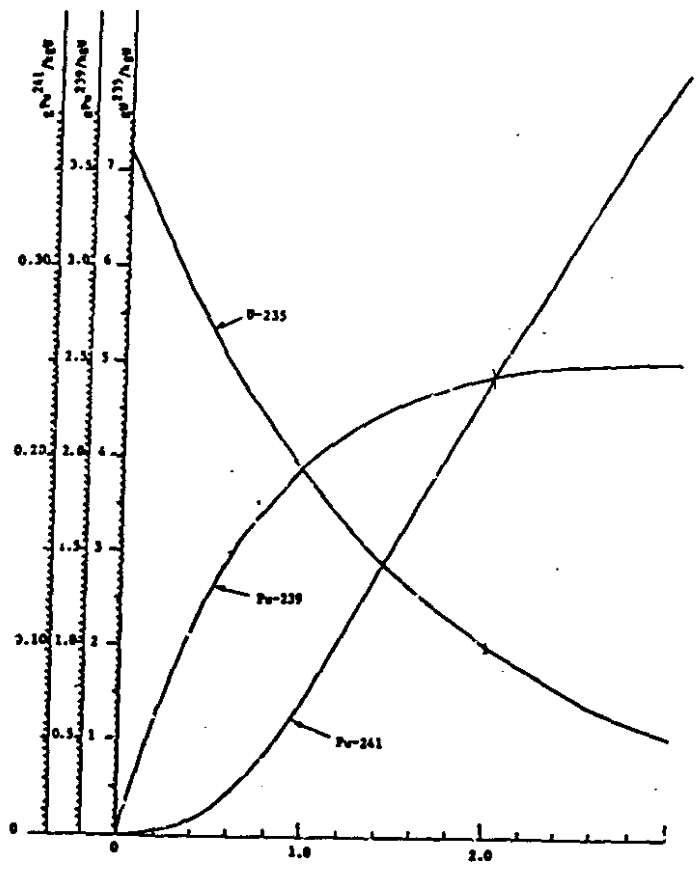
Figure 7.1 Fissile isotope concentrations as a function of burnup

TABLE 1

Burnup Data*

n/kb	MWh/kgU	U-235 (g/kgU)	Pu-239 (g/kgU)	Pu-241 (g/kgU)
0	0	7.20	0	0
0.2	19	6.37	0.60	0.002
0.4	39	5.62	1.10	0.009
0.6	59	4.90	1.48	0.025
0.8	79	4.30	1.77	0.049
1.0	100	3.76	1.98	0.078
1.2	120	3.32	2.14	0.107
1.4	140	2.90	2.25	0.145
1.6	159	2.56	2.33	0.177
1.8	179	2.26	2.39	0.211
2.0	198	1.98	2.43	0.245
2.2	216	1.74	2.46	0.278
2.4	235	1.54	2.48	0.309
2.6	253	1.35	2.49	0.338
2.8	271	1.18	2.50	0.366
3.0	289	1.03	2.50	0.393

* The values shown in this table strictly speaking apply only to the Pickering reactors, but they will be correct to within a percent or so for all natural uranium, D_2O moderated reactors.



QUESTION : Specific fissile content of fuel as a function of burnup.

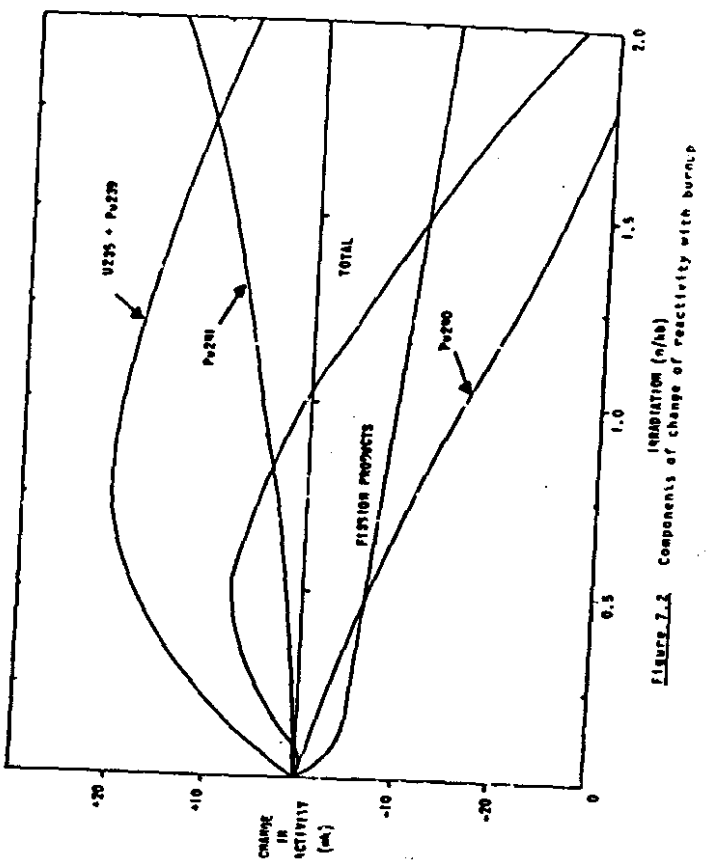


Figure 7.2 Components of change of reactivity with burnup

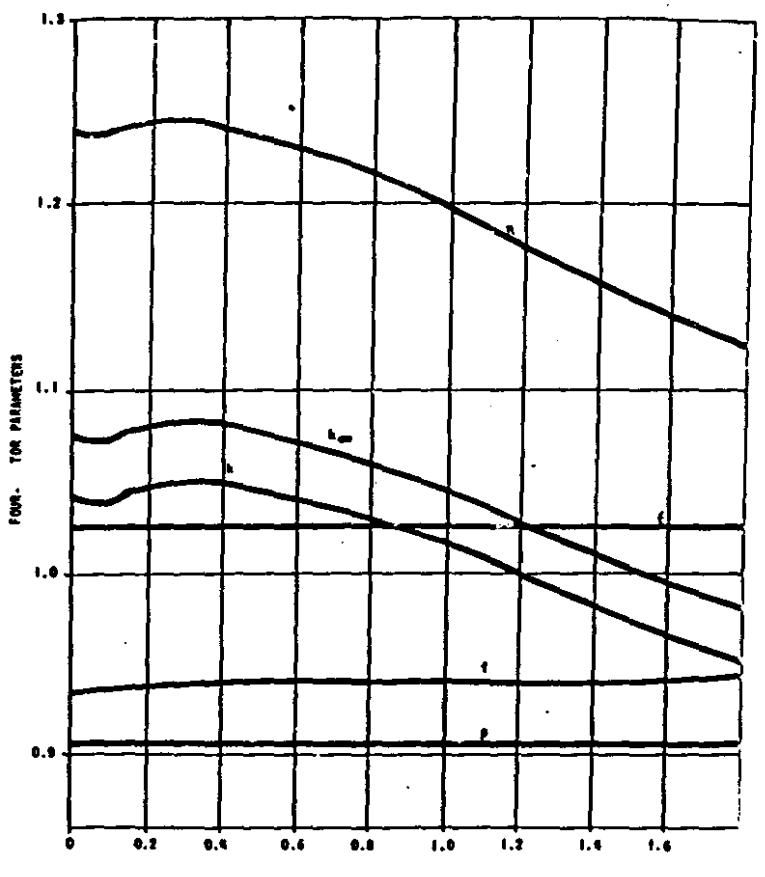
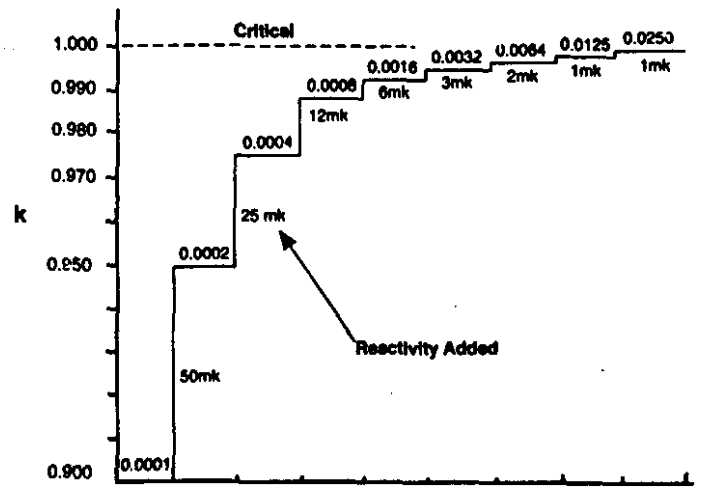


Figure 7.3 Variation of four-factor parameters, k_∞ and k with burnup

Approach to Critical Multiplication Factor



OH 14.2

Approach to Critical

$$P_\infty = \frac{P_0}{1-k} = -\frac{P_0}{\Delta k}$$

- Assume that the reactor has been shutdown for a few days with -100mk of reactivity
- Assume photoneutron power = 0.001% = 0.00001

$$P_\infty = \frac{0.00001}{0.1} = 0.0001$$

- Assume power P_∞ is doubled at each step

P_0	P_∞	k	Required Δk	P_0/P_∞	$1/c = 1/S_\infty$	Δk Added
0.00001	0.0001	0.900	0.100	0.1	10,000	0
0.00001	0.0002	0.950	0.050	0.05	5,000	50
0.00001	0.0004	0.975	0.025	0.025	2,500	25
0.00001	0.0008	0.988	0.012	0.0125	1,250	12
0.00001	0.0016	0.994	0.006	0.0062	625	6
0.00001	0.0032	0.997	0.003	0.0031	312	3
0.00001	0.0064	0.998	0.002	0.0016	156	2
0.00001	0.0125	0.999	0.001	0.0008	78	1
0.00001	0.0250	1.000	0.000	0.0004	40	1

Reactor Goes "Critical" At 2.5% Full Power
S_0 Is Only 0.04% Of Total Neutron Population

OH 14.1

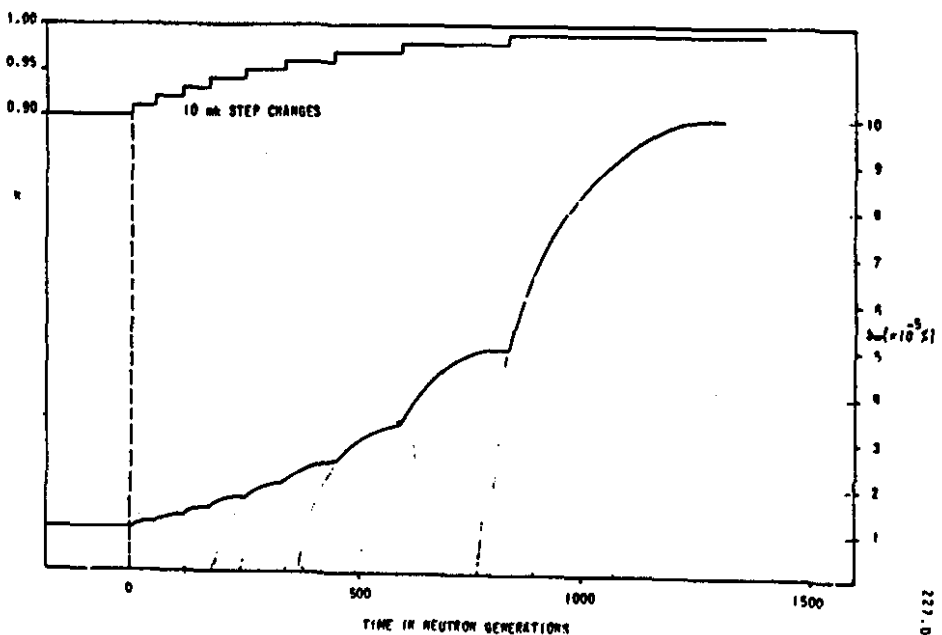
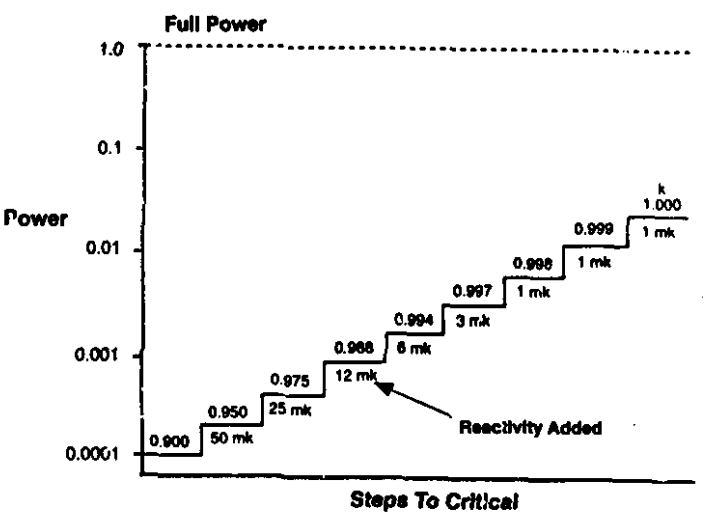
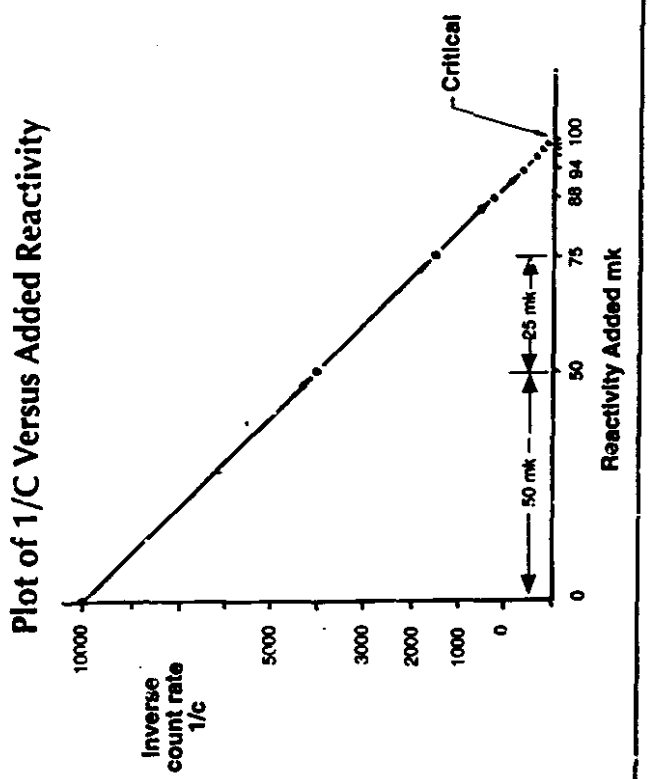


Figure 9.1 Stabilization time after change in multiplication factor

Approach to Critical – Power Level



OH 14.3



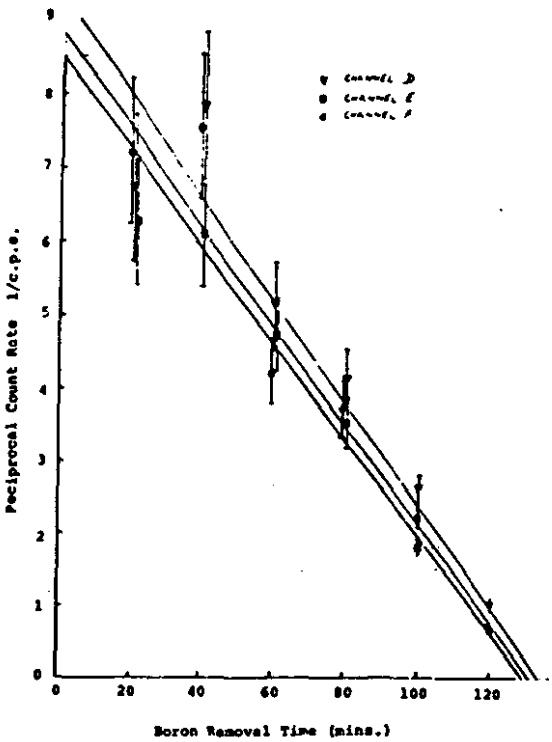


Fig. 8 Approach-to-Critical Graph (Pickering Unit 3)

Approach From -500mk

Assume that the reactor has been shutdown for a few days with -500mk of reactivity

Assume photoneutron power = 0.00001 as before

P_0	P_-	k	Required Δk	P_0/P_-
0.00001	0.00002	0.500	0.500	0.5
0.00001	0.00008	0.875	0.125	0.125
0.00001	0.00032	0.967	0.031	0.031
0.00001	0.00125	0.992	0.008	0.008
0.00001	0.00500	0.998	0.002	0.002
0.00001	0.02000	1.000	0.000	0.000

Reactor goes "critical" at 2% full power if last line is not rounded off then:

0.00001	0.01000	0.999	0.001	0.001
0.00001	0.02	0.9995	0.0005	0.0005
0.00001	0.08	0.999875	0.000125	0.000125
0.00001	0.32	0.999969	0.000031	0.000031
0.00001	0.64	0.999984	0.000015	0.000016

At 64% Power k = 0.999984

Approach After 3 Months

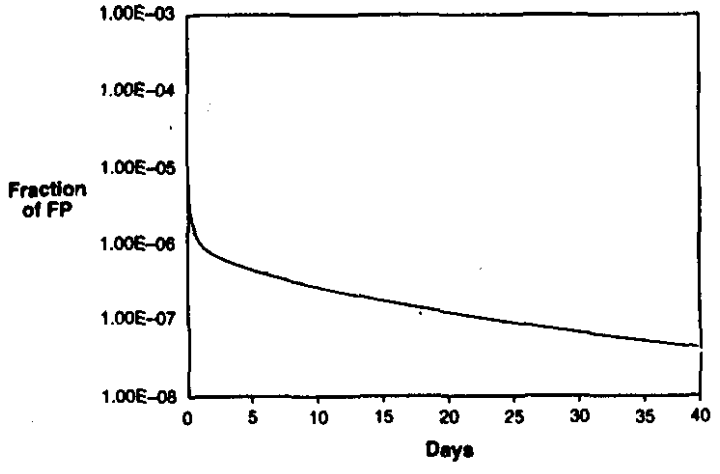
Assume that the reactor has been shutdown for 3 months with -100mk of reactivity

Assume photoneutron power = 0.00001% = 0.0000001

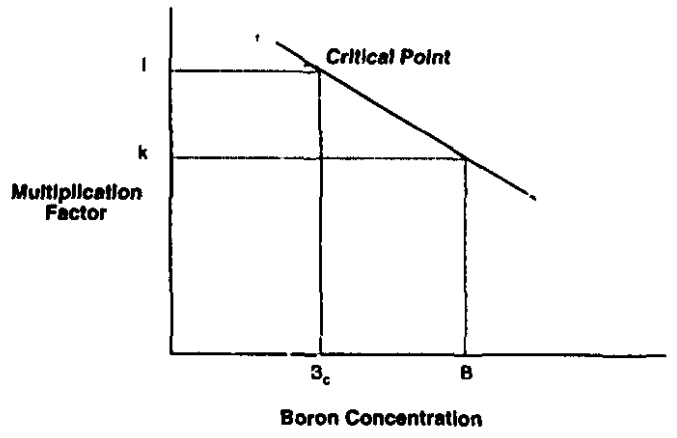
P_0	P_-	k	Required Δk	P_0/P_-
0.0000001	0.000001	0.900	0.100	0.1
0.0000001	0.000004	0.975	0.025	0.025
0.0000001	0.000016	0.994	0.006	0.0062
0.0000001	0.000064	0.998	0.002	0.0016
0.0000001	0.000250	1.000	0.000	0.0004

Reactor goes "critical" at 0.025% full power

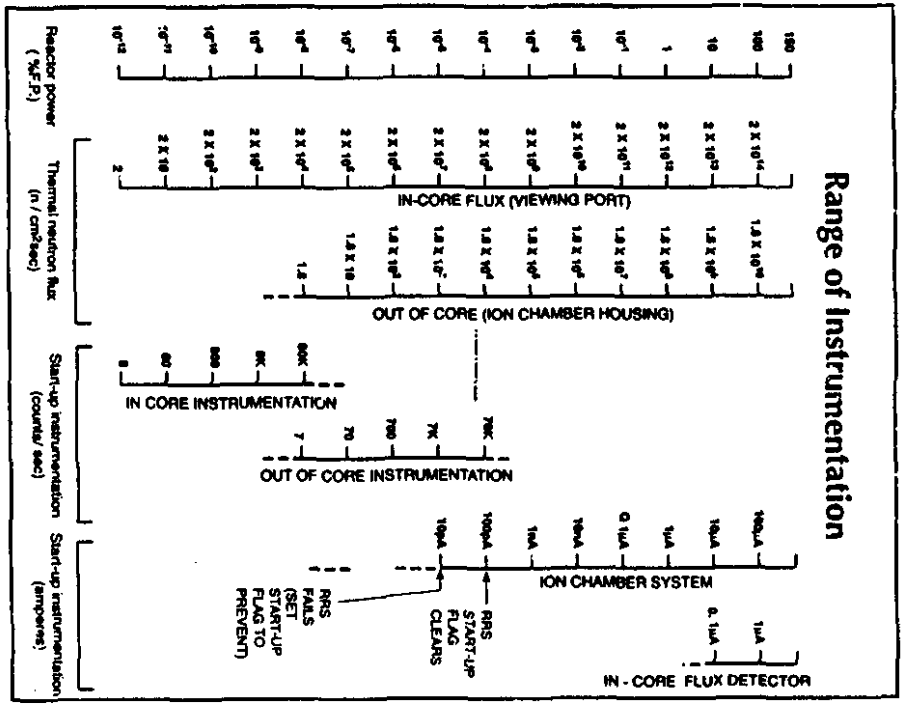
Flux Level After Shutdown
Reactor 400 mk Subcritical



Multiplication Factor Versus Poison Concentration



Range of Instrumentation



Plot of Inverse Count Rate Versus Poison Load

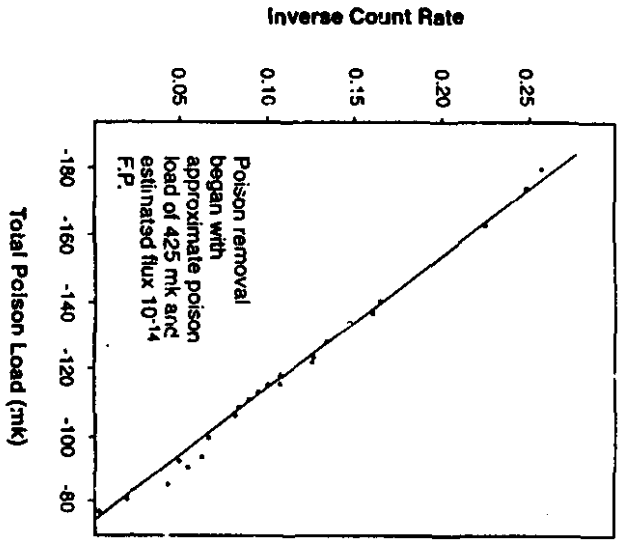


Fig. 14.2